Multi-scale (Coastal Ocean) Data Assimilation

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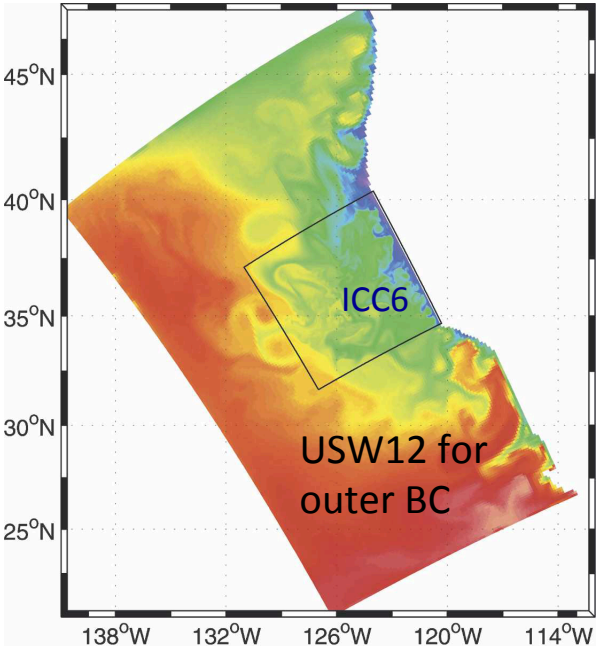


Motivation I: Multi-Scaleness in the Model State

- As the model advances & becomes capable of complex physical processes, ocean dynamics exhibits multi-scale phenomena.

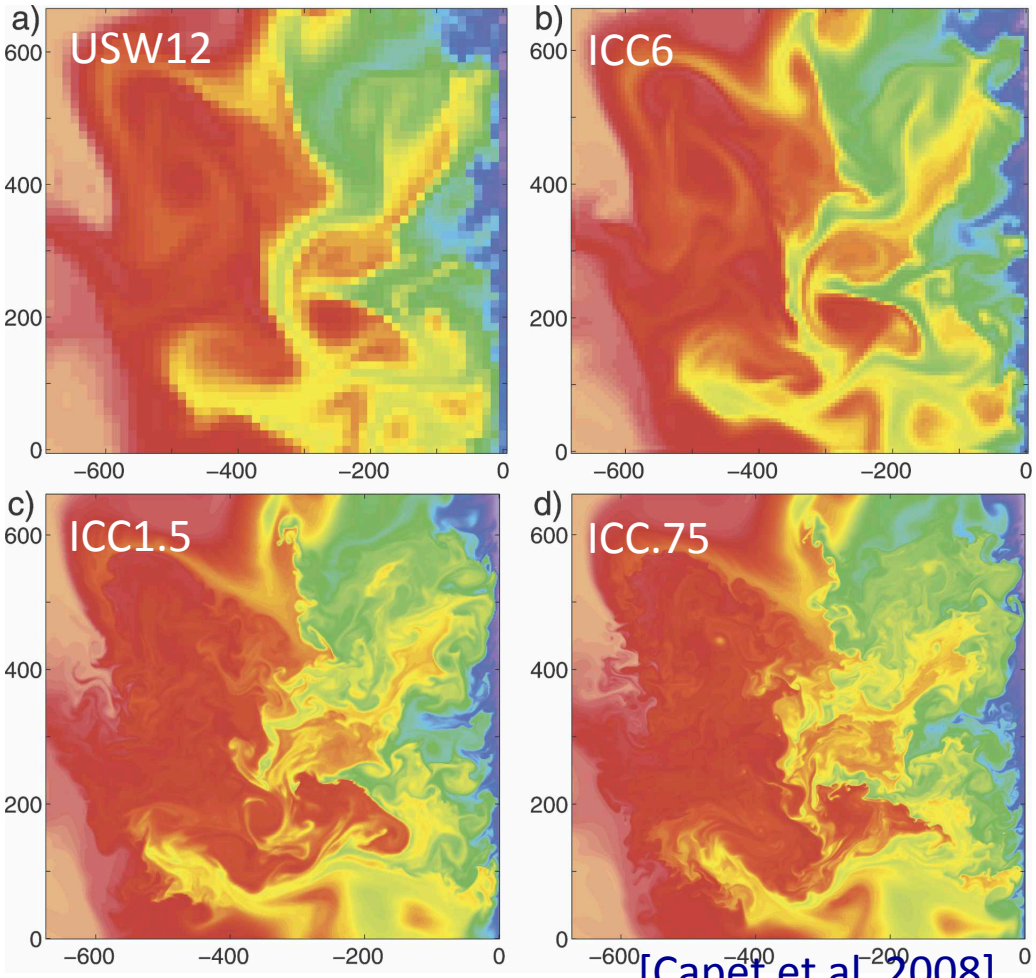
  - High-resolution model state

$$\mathbf{x} = \mathbf{x}_L + \mathbf{x}_S$$

$\mathbf{x}_L$: large-scale

$\mathbf{x}_S$: small-scale

  - This work focuses on spatial multi-scale/multi-resolution in horizontal



[Capet et al, 2008]

Multi-Scale Incremental Cost Function for **x**

- Original incremental cost function

$$J(\Delta\mathbf{x}) = \frac{1}{2}\Delta\mathbf{x}^T\mathbf{B}^{-1}\Delta\mathbf{x} + \frac{1}{2}(\mathbf{H}\Delta\mathbf{x}-\mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\Delta\mathbf{x}-\mathbf{d})$$

  - Control variable: $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}^b$ (departure from $\mathbf{x}^b$)
  - Innovation: $\mathbf{d} = \mathbf{y}^o - \mathbf{H}\mathbf{x}^b$ ($\mathbf{H}$: $\mathbf{x}\rightarrow\mathbf{y}$ is the obs. operator)
  - **B** and **R**: background and observation error covariance matrices

- Multi-scale cost function for $\Delta\mathbf{x}=\Delta\mathbf{x}_L+\Delta\mathbf{x}_S$ with $\mathbf{B}=\mathbf{B}_L+\mathbf{B}_S$ [Wu et al, 2002]

$$J(\Delta\mathbf{x}_L,\Delta\mathbf{x}_S) = \frac{1}{2}(\Delta\mathbf{x}_L)^T\mathbf{B}_L^{-1}\Delta\mathbf{x}_L + \frac{1}{2}(\Delta\mathbf{x}_S)^T\mathbf{B}_S^{-1}\Delta\mathbf{x}_S + \frac{1}{2}(\mathbf{H}\Delta\mathbf{x}-\mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\Delta\mathbf{x}-\mathbf{d})$$

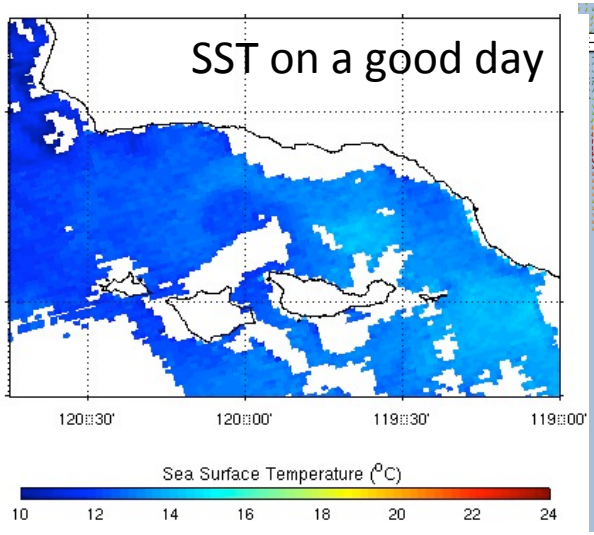
- Revised & equivalent cost functions by decoupling the scales:

$$J_L(\Delta\mathbf{x}_L) = \frac{1}{2}(\Delta\mathbf{x}_L)^T\mathbf{B}_L^{-1}\Delta\mathbf{x}_L + \frac{1}{2}(\mathbf{H}\Delta\mathbf{x}_L-\mathbf{d})^T(\mathbf{R}+\mathbf{H}\mathbf{B}_S\mathbf{H}^T)^{-1}(\mathbf{H}\Delta\mathbf{x}_L-\mathbf{d})$$

$$J_S(\Delta\mathbf{x}_S) = \frac{1}{2}(\Delta\mathbf{x}_S)^T\mathbf{B}_S^{-1}\Delta\mathbf{x}_S + \frac{1}{2}(\mathbf{H}\Delta\mathbf{x}_S-\mathbf{d})^T(\mathbf{R}+\mathbf{H}\mathbf{B}_L\mathbf{H}^T)^{-1}(\mathbf{H}\Delta\mathbf{x}_S-\mathbf{d})$$

Motivation II: Ocean Observation Networks

- Spatial distribution of observing networks is highly inhomogeneous, for example
  - Satellite images (SST) can be as high-resolution as the model state in horizontal at surface
  - HR radar (surface velocity) can be highly concentrated and high-resolution
  - Others can be extremely sparse



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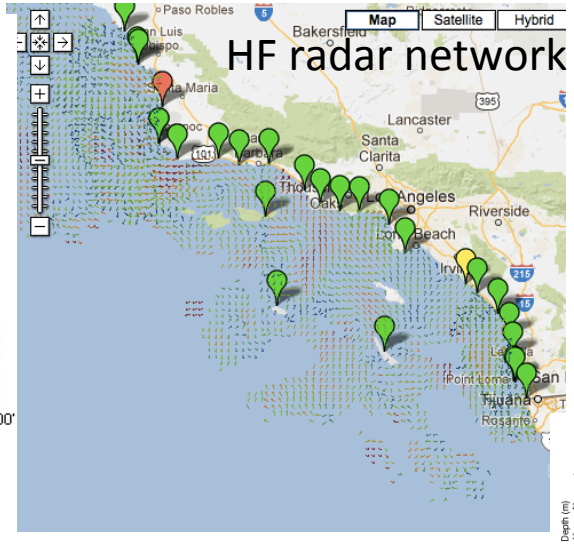
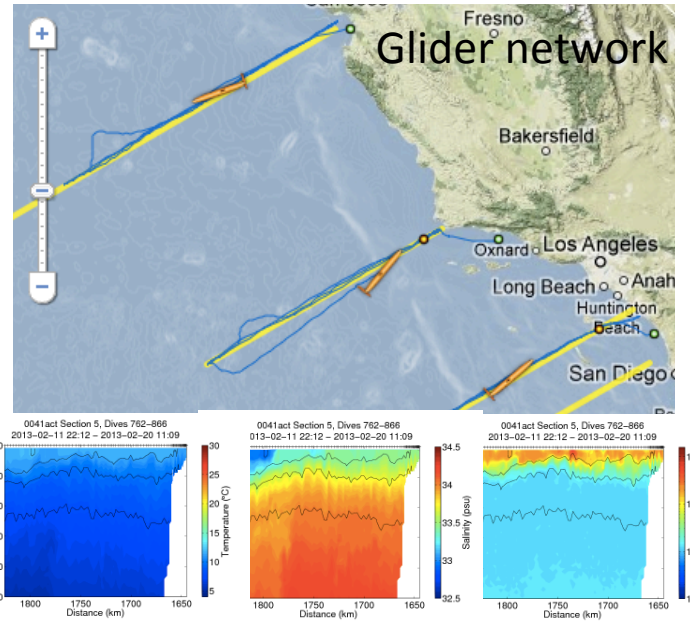




Multi-Scale Cost Function for **x** and **y**

- Observation classification based on network resolution

$$\mathbf{y}=\begin{pmatrix}\mathbf{y}_D\\ \mathbf{y}_C\end{pmatrix}=\begin{pmatrix}\mathbf{H}_D(\mathbf{x}_L+\mathbf{x}_S)\\ \mathbf{H}_C(\mathbf{x}_L+\mathbf{x}_S)\end{pmatrix}=\begin{pmatrix}\text{dense}\\ \text{coarse}\end{pmatrix}\qquad \text{with}\quad \begin{pmatrix}\mathbf{R}_D\\ \mathbf{R}_C\end{pmatrix}$$

- Multi-scale decomposition for $\mathbf{y}_D$ by smoothing

$$\mathbf{y}^o=\begin{pmatrix}\mathbf{y}^o_D\\ \mathbf{y}^o_C\end{pmatrix}=\begin{pmatrix}\mathbf{y}^o_{D.L}+\mathbf{y}^o_{D.S}\\ \mathbf{y}^o_C\end{pmatrix}\qquad \Rightarrow \mathbf{y}=\begin{pmatrix}\mathbf{y}_{D.L}\\ \mathbf{y}_{D.S}\\ \mathbf{y}_C\end{pmatrix}=\begin{pmatrix}\mathbf{H}_D\mathbf{x}_L\\ \mathbf{H}_D\mathbf{x}_S\\ \mathbf{H}_C(\mathbf{x}_L+\mathbf{x}_S)\end{pmatrix}\quad \text{with}\quad \begin{pmatrix}\mathbf{R}_{D.LL}\\ \mathbf{R}_{D.SS}\\ \mathbf{R}_C\end{pmatrix}$$

leading to MS 3D-Var

$$J_L(\Delta\mathbf{x}_L)=\frac{1}{2}(\Delta\mathbf{x}_L)^T\mathbf{B}_L^{-1}\Delta\mathbf{x}_L+\frac{1}{2}(\mathbf{H}_D\Delta\mathbf{x}_L-\mathbf{d}_{D.L})^T(\mathbf{R}_{D.L})^{-1}(\mathbf{H}_D\Delta\mathbf{x}_L-\mathbf{d}_{D.L})$$
$$+\frac{1}{2}(\mathbf{H}_C\Delta\mathbf{x}_L-\mathbf{d}_C)^T(\mathbf{R}_C+\mathbf{H}_C\mathbf{B}_S\mathbf{H}_C^T)^{-1}(\mathbf{H}_C\Delta\mathbf{x}_L-\mathbf{d}_C)$$

$$J_S(\Delta\mathbf{x}_S)=\frac{1}{2}(\Delta\mathbf{x}_S)^T\mathbf{B}_S^{-1}\Delta\mathbf{x}_S+\frac{1}{2}(\mathbf{H}_D\Delta\mathbf{x}_S-\mathbf{d}_{D.S})^T(\mathbf{R}_{D.S})^{-1}(\mathbf{H}_D\Delta\mathbf{x}_S-\mathbf{d}_{D.S})$$
$$+\frac{1}{2}(\mathbf{H}_C\Delta\mathbf{x}_S-\mathbf{d}_C)^T(\mathbf{R}_C+\mathbf{H}_C\mathbf{B}_L\mathbf{H}_C^T)^{-1}(\mathbf{H}_C\Delta\mathbf{x}_S-\mathbf{d}_C)$$

Practical Implementation of MS 3D-Var

- Sequential Implementation
  - Estimation of dynamically important Large-scale first, then higher density to capture smaller scales
  - Similar to Successive Covariance Localization (SCL: Zhang et al, 2009)

    →Use of $\mathbf{B}=\mathbf{B}_L+\mathbf{B}_S$: Additive Background Covariance (AB 3D-Var)
  - Explicit separation in $\mathbf{y}=\mathbf{y}_{D.L}+\mathbf{y}_{D.S}$

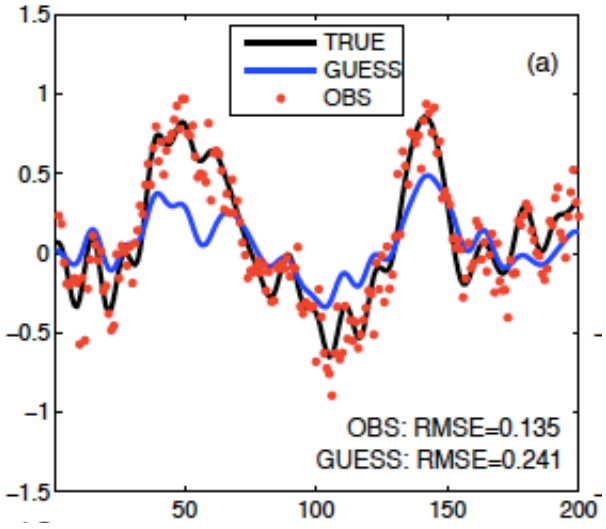
- MS 3D-Var:
  - Large-scale $$J_L(\Delta\mathbf{x}_L)=\frac{1}{2}(\Delta\mathbf{x}_L)^T\mathbf{B}_L^{-1}\Delta\mathbf{x}_L+\frac{1}{2}(\mathbf{H}_D\Delta\mathbf{x}_L-\mathbf{d}_{D.L})^T(\mathbf{R}_{D.L})^{-1}(\mathbf{H}_D\Delta\mathbf{x}_L-\mathbf{d}_{D.L})$$
  $$+\frac{1}{2}(\mathbf{H}_C\Delta\mathbf{x}_L-\mathbf{d}_C)^T(\mathbf{R}_C+\mathbf{H}_C\mathbf{B}_S\mathbf{H}_C^T)^{-1}(\mathbf{H}_C\Delta\mathbf{x}_L-\mathbf{d}_C$$
  - Small-scale $$J_S(\Delta\mathbf{x}_S)=\frac{1}{2}(\Delta\mathbf{x}_S)^T\mathbf{B}_S^{-1}\Delta\mathbf{x}_S+\frac{1}{2}(\mathbf{H}_D\Delta\mathbf{x}_S-\mathbf{d}_{D.S}^{a.L})^T(\mathbf{R}_{D.S})^{-1}(\mathbf{H}_D\Delta\mathbf{x}_S-\mathbf{d}_{D.S}^{a.L})$$
  $$+\frac{1}{2}(\mathbf{H}_C\Delta\mathbf{x}_S-\mathbf{d}_C)^T(\mathbf{R}_C+\mathbf{H}_C\mathbf{B}_L\mathbf{H}_C^T)^{-1}(\mathbf{H}_C\Delta\mathbf{x}_S-\mathbf{d}_C)$$

$$\mathbf{d}_{D.S}^a=\mathbf{d}_{D.S}-\mathbf{H}_D\Delta\mathbf{x}_L^a$$

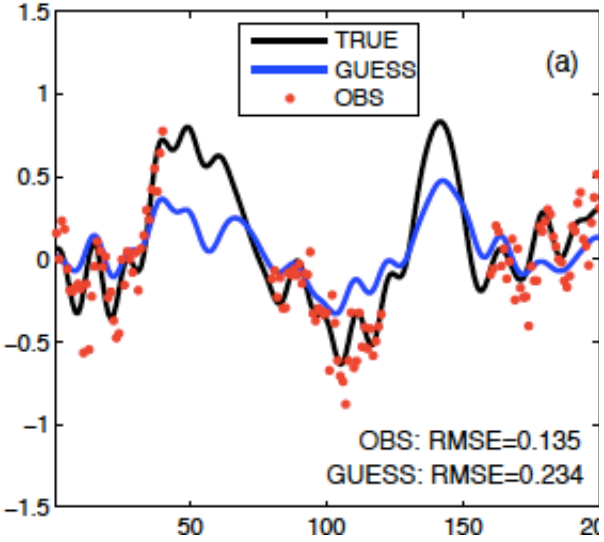
Demonstration: Simple Demonstration (1D-Var)

- MS Data Assimilation System
  - **x**:
    - $\mathbf{x}^t$ is MS $\quad x_n^t = S_0 \sum_{k=1}^{K} a_k^t \cos\left(\frac{k\pi n}{N} + \phi_n^t\right): \qquad a_k^t = k^{-\gamma}$ with $\gamma \subset [0,2]$
    - $\mathbf{x}^b$ is MS $\quad x_n^b = S_0 \sum_{k=1}^{K} a_k^b \cos\left(\frac{k\pi n}{N} + \phi_n^b\right): \qquad a_k^b = p_0 \beta_k a_k^t$ with $\beta_k \subset U(0,1)$
    - **B** may be MS with $(D_L, D_S)=(40, 5)$ & properly estimated (σ^b_L, σ^b_S)
      may be SS with $D=40, 20, 10, 5$ & properly σ^b
  - **y**=**Hx** network:

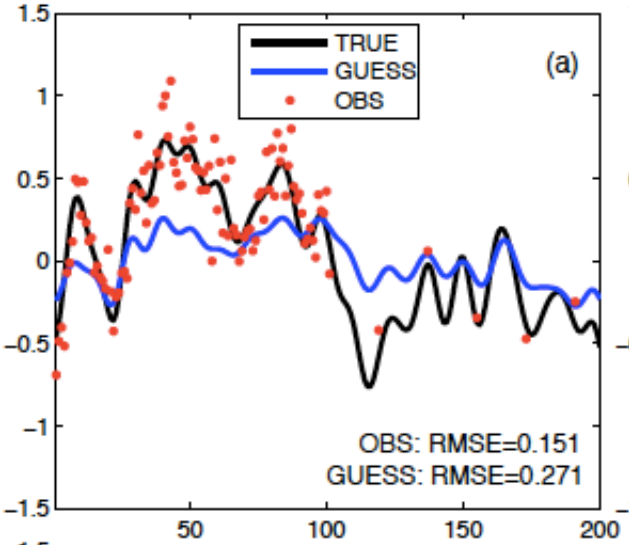
completely dense



patchy dense

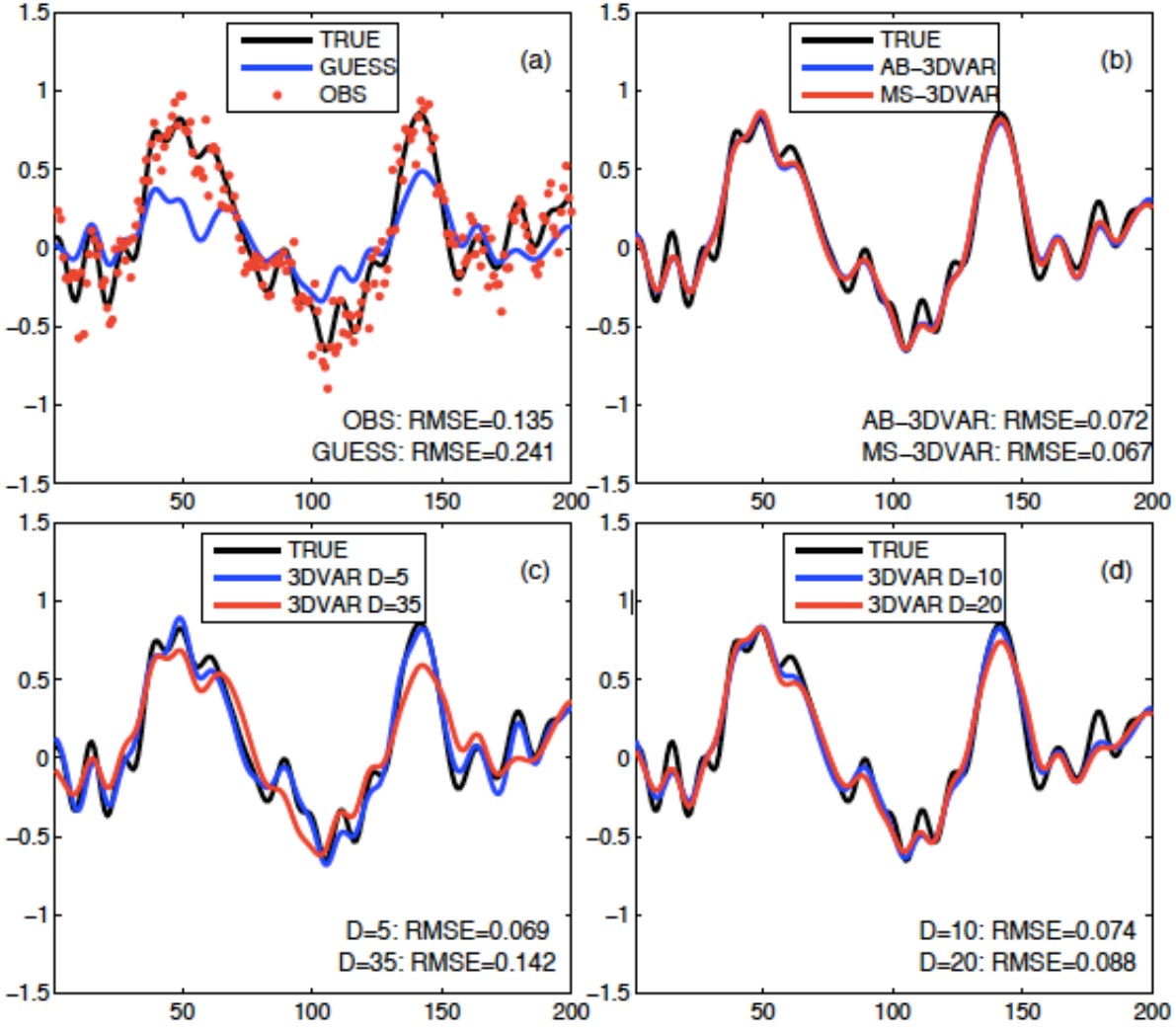


mixed dense-coarse



Completely Dense Observing Network

- Analysis



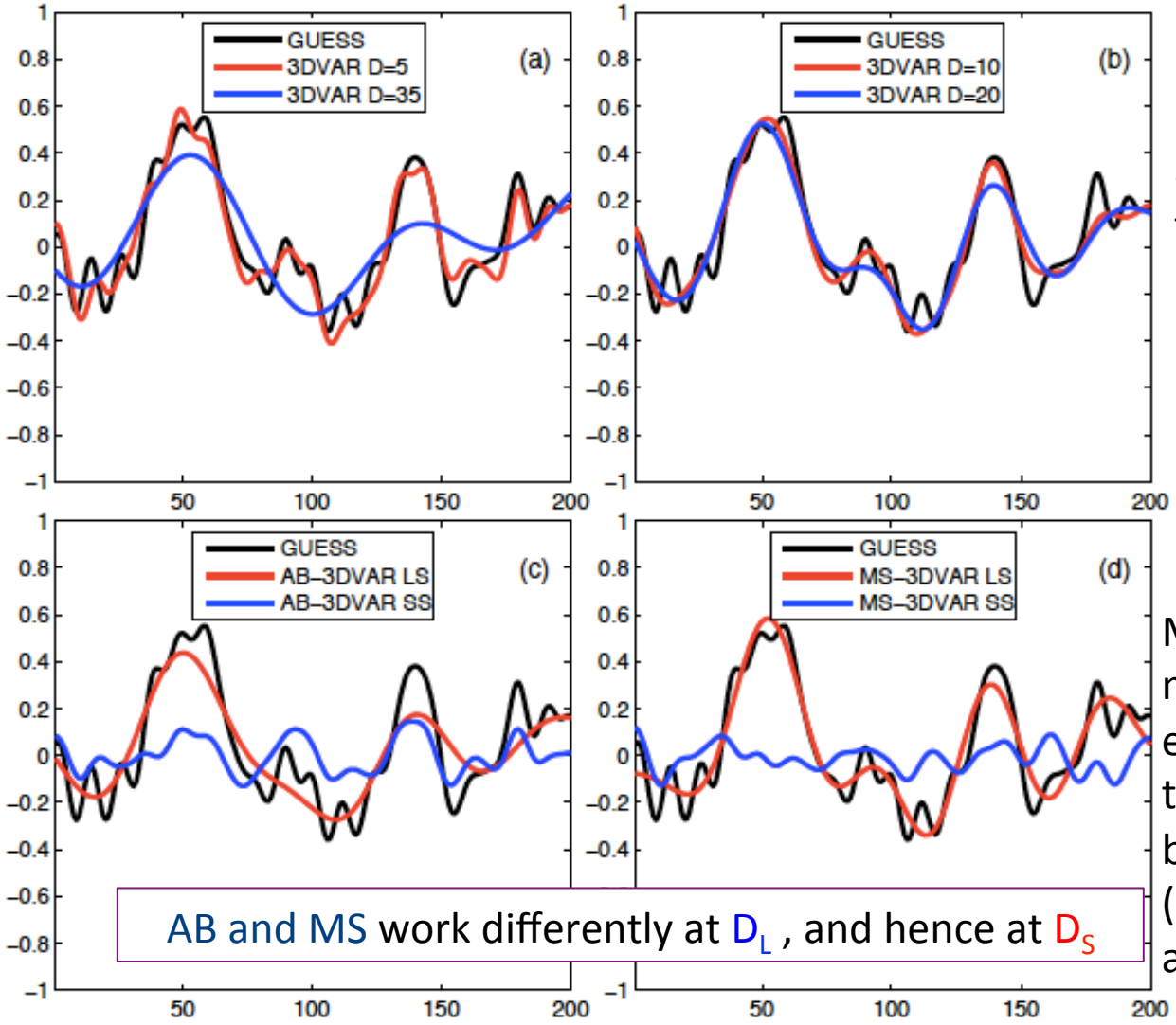
AB and MS work we with D_S & D_L

3V work OK at D_S not D_L

3V work OK at medium D

Completely Dense Observing Network

- Analysis Increment

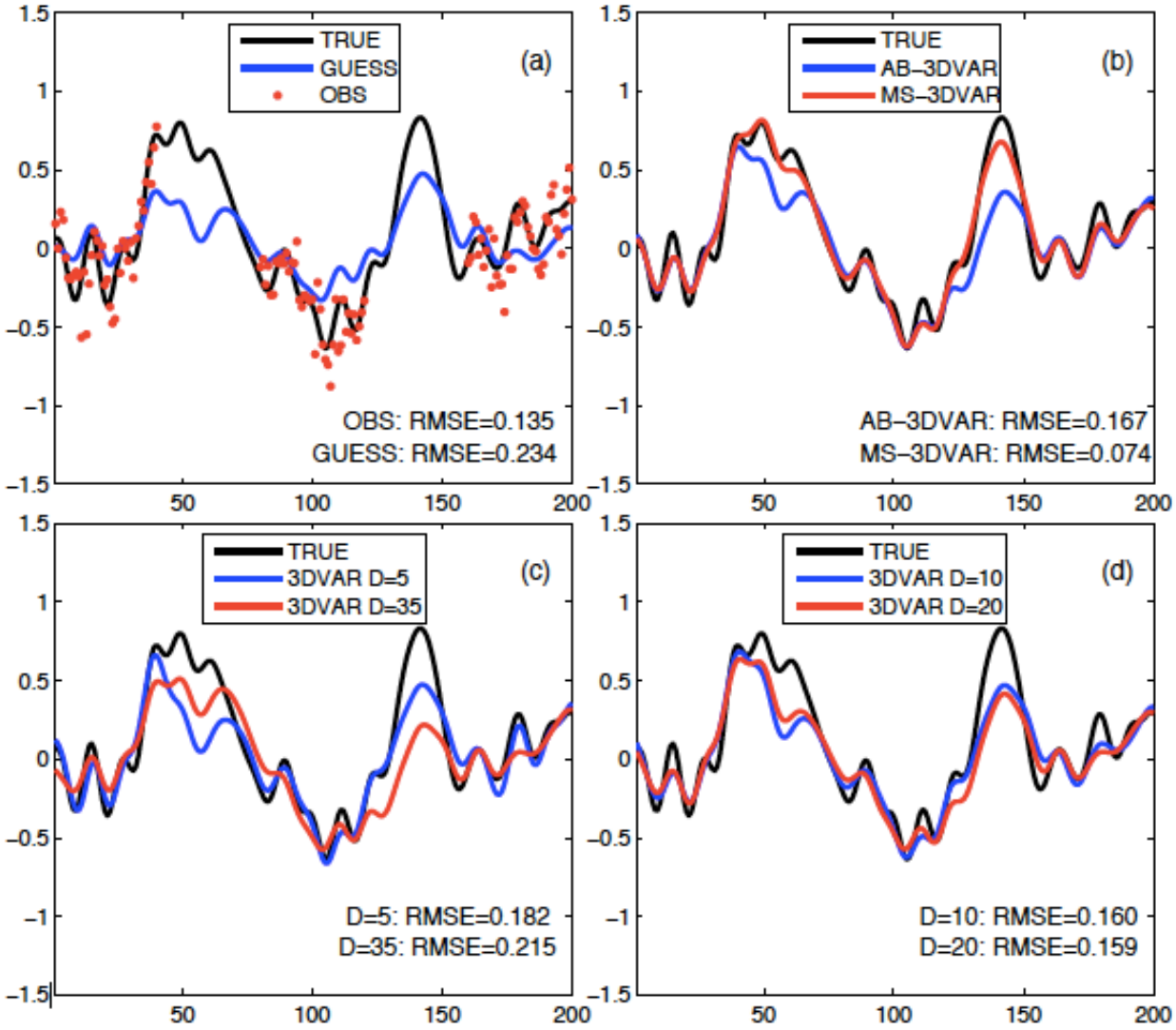


3V work better at medium D: than D_S or D_L

MS captures LS more effectively than AB by sequential (successive) approach

Patchy Dense Observation Network

- Analysis



MS works much better than AB

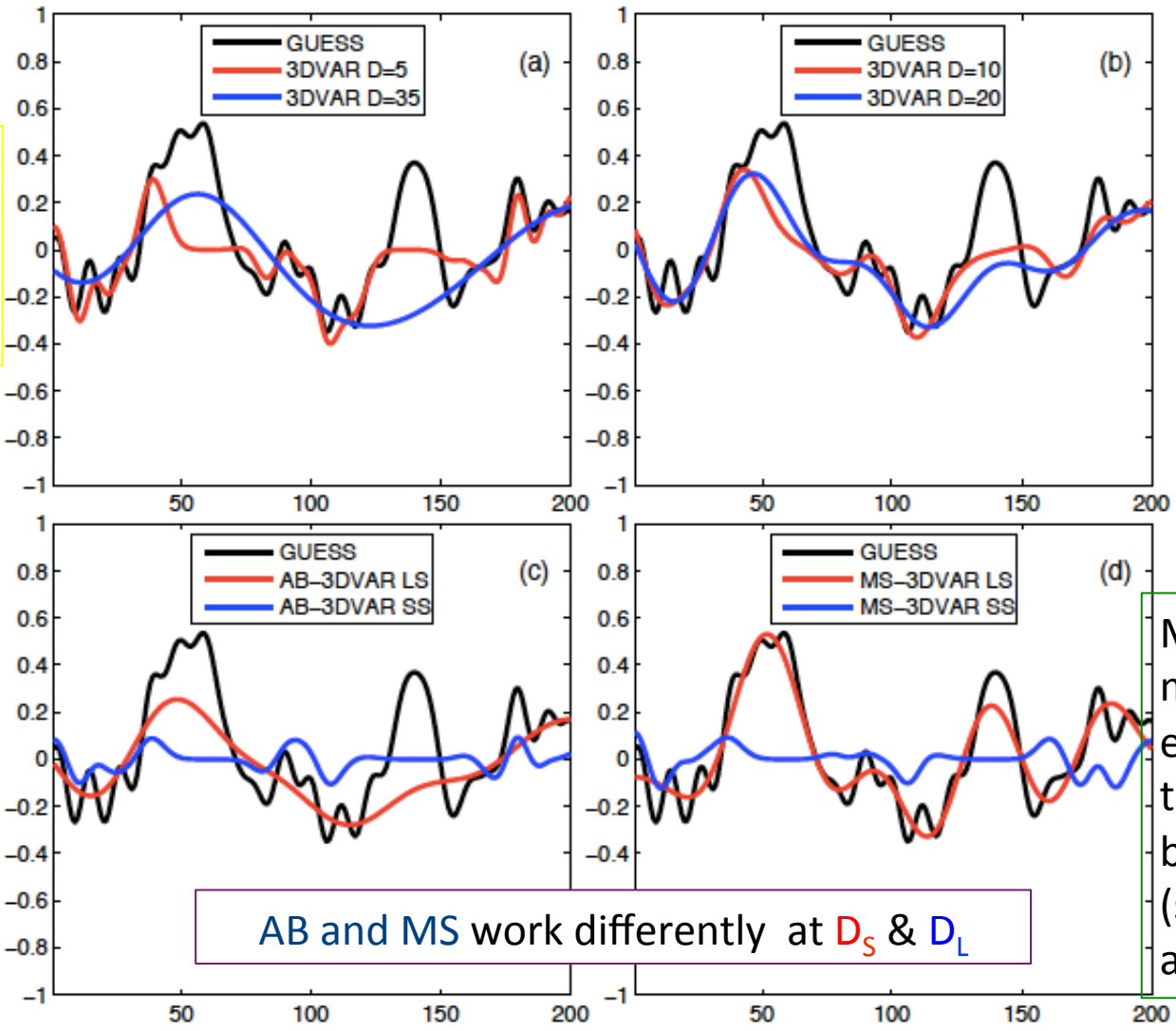
3V don't work well: Better at D_S than D_L

3V work OK at medium D

Patchy Dense Observation Network

- Analysis increment

3V don't work well: Better at D_S than D_L

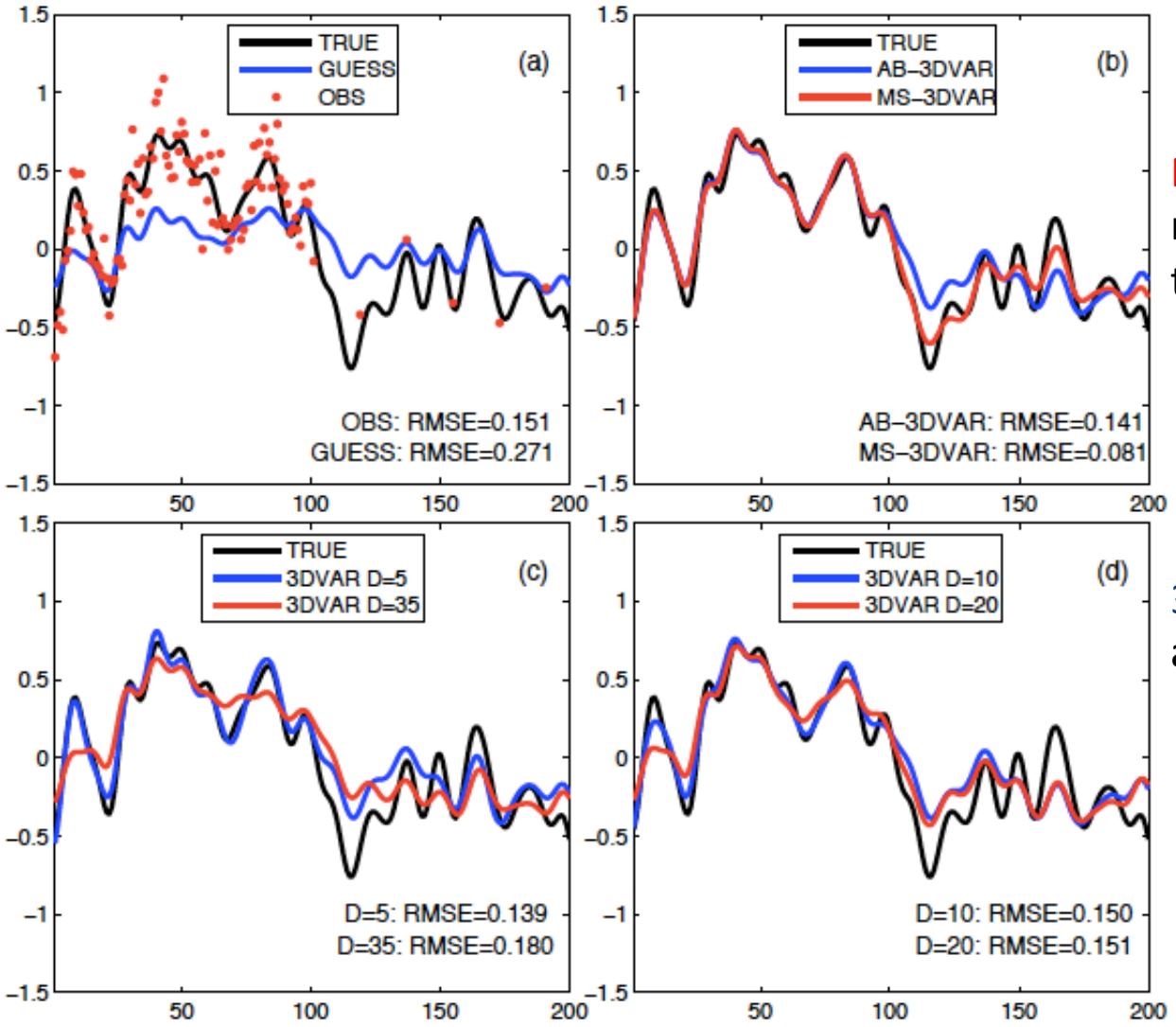


AB and MS work differently at D_S & D_L

MS captures LS more effectively than AB by sequential (successive) approach

Mixed Resolution Network

- Analysis



MS works much better than AB

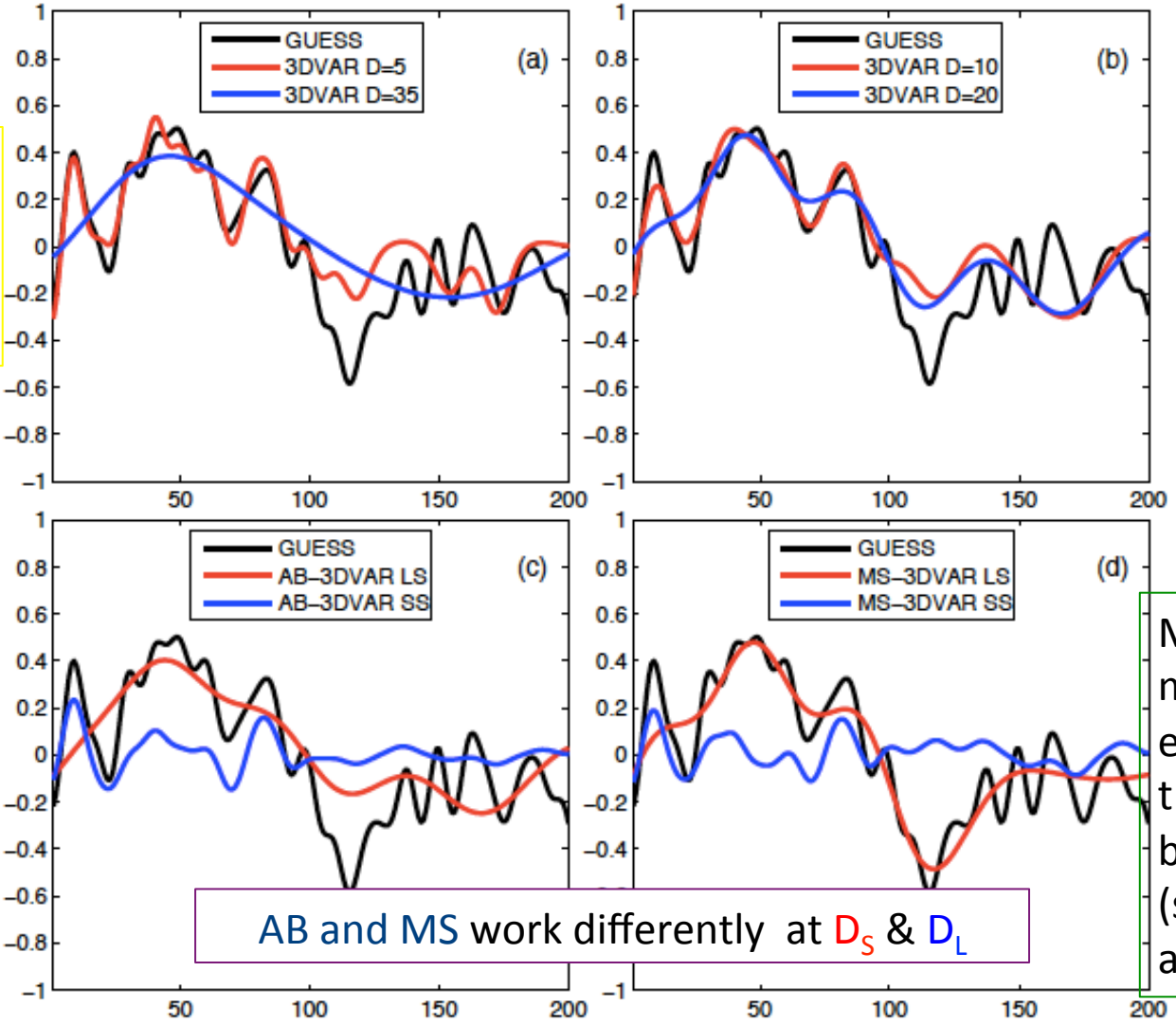
3V work OK: Better at D_S than D_L

3V work OK at medium D

Mixed Resolution Network

- Analysis increment

3V don't work well: Better at D_S than D_L



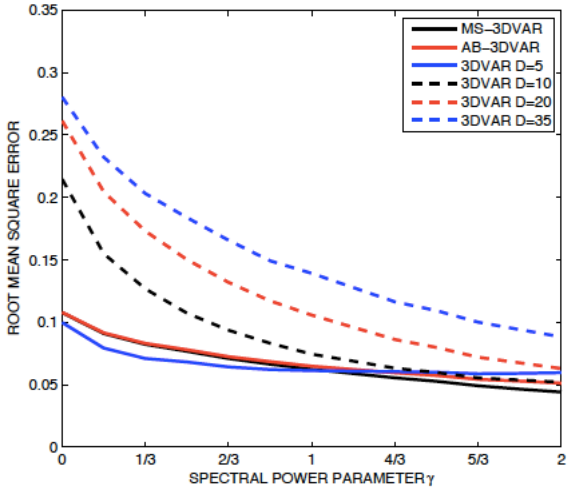
AB and MS work differently at D_S & D_L

MS captures LS more effectively than AB by sequential (successive) approach

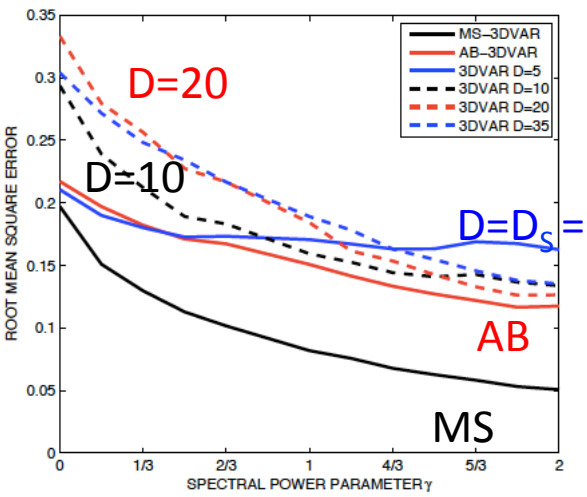
RMSE Error: Scales

- RMSE error in scales

completely dense



patchy dense



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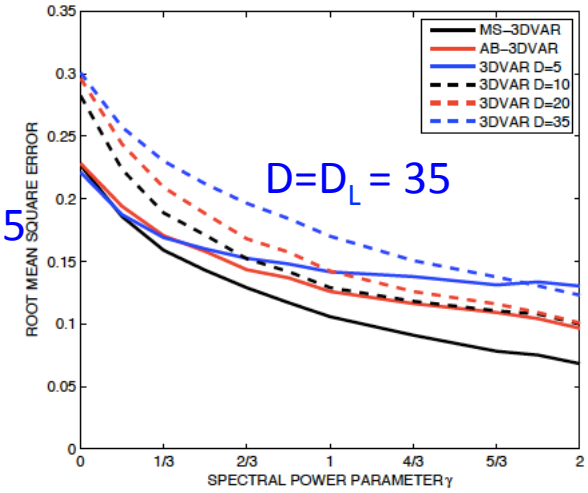
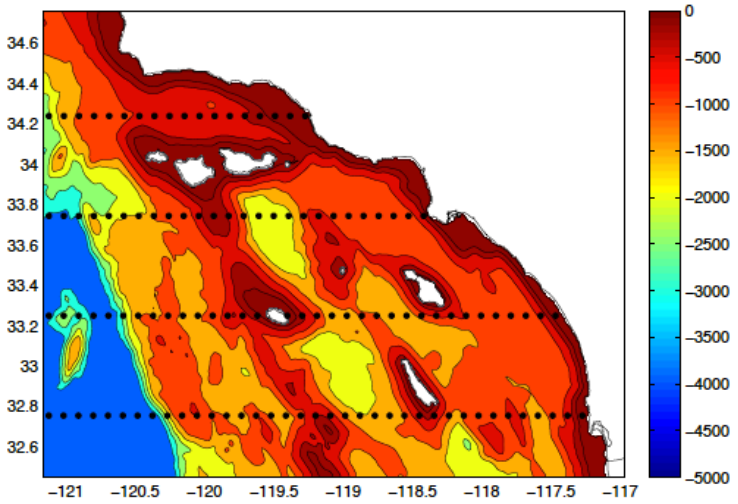


California Coastal Ocean Data Assimilation System

- Southern California Bight (SCB) Coastal Ocean Modeling System
  - Regional Ocean Modeling System (ROMS)
    - Resolution: 1km x 40 levels nested in low-resolution model
    - Atmos forcing: WRF at 2km
  - Southern California Coastal Ocean Observing System (SCCOS)

- OSSE Observing network
  - SST at 2km resolution
  - Surface (u,v) at 2km resolution
  - T/S profiling along 4 tracks at
    - 60km<D_L separation between tracks
    - 10km<D_s, D_{3Dvar} along track

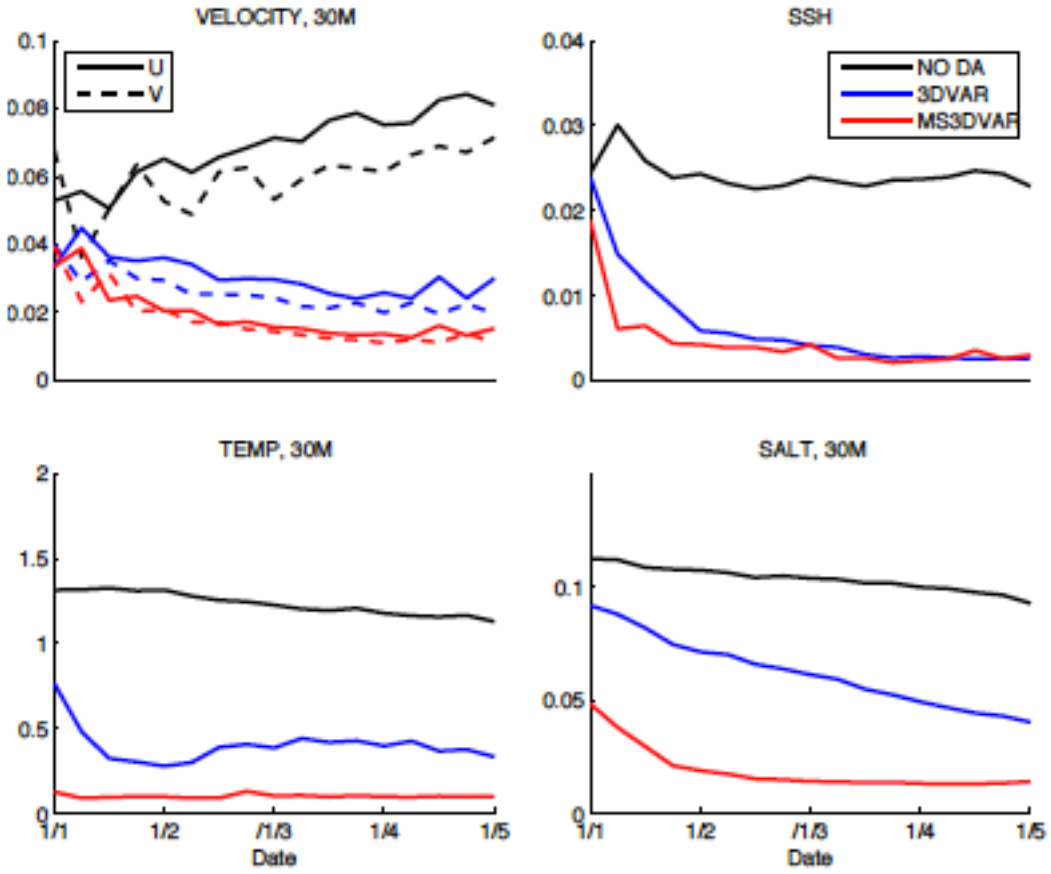
    Up to 400m



Bathymetry and T/S profiling positions

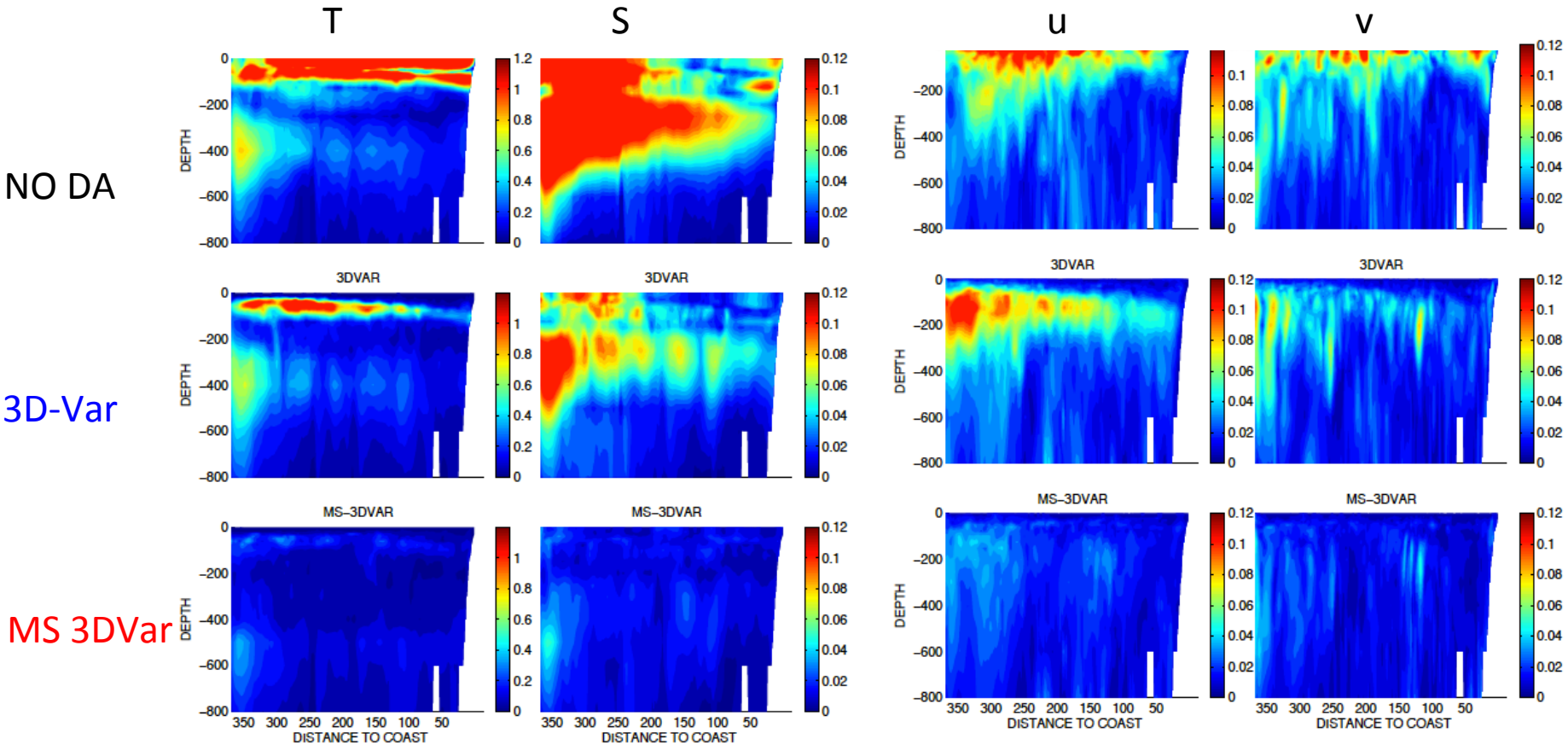
OSSE: RMSE in Time

- Comparison between
  - NO DA
  - Standard 3D-Var
  - MS 3D-Var
    - Converges faster and smaller RMSE



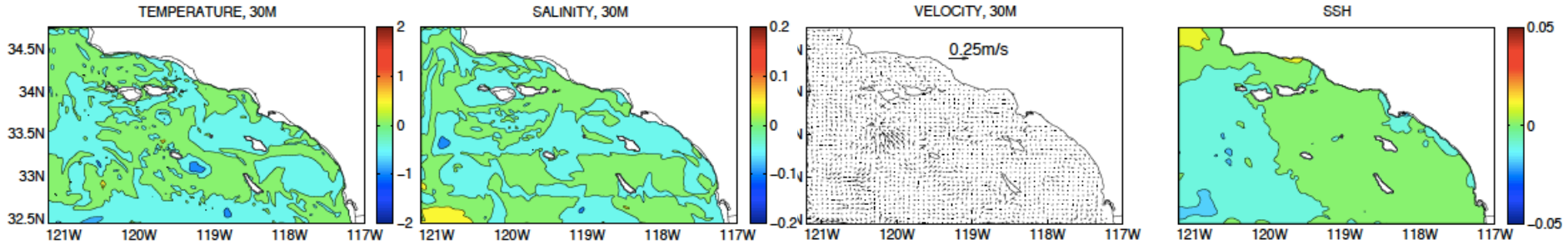
OSSE: RMSE Alongshore

- At Day 3

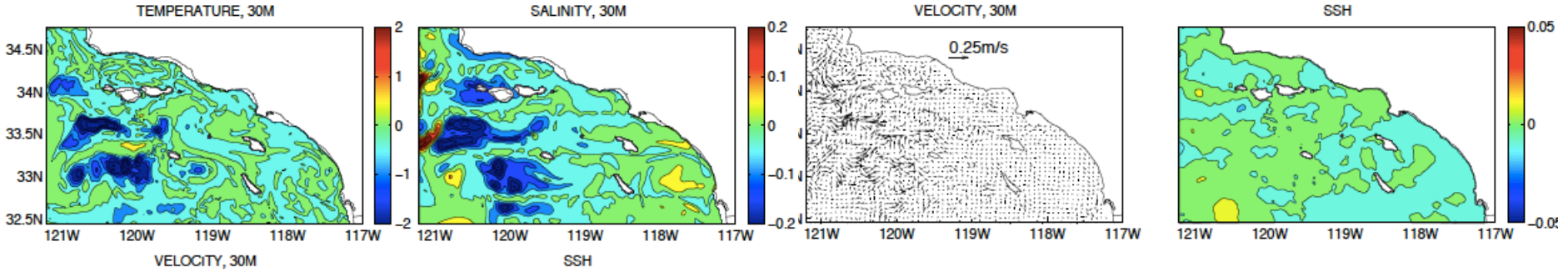


OSSE: Instantaneous Error. z=30m, day 4

- MS 3D-Var

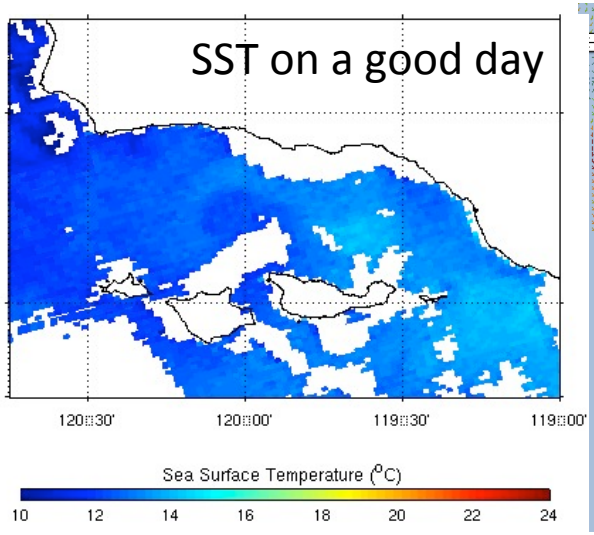


- Standard 3D-Var



Real Observation Experiments

- Initialization: 01.01.08
  - No DA
  - Standard 3D-Var
  - MS 3D-Var
- Observing system



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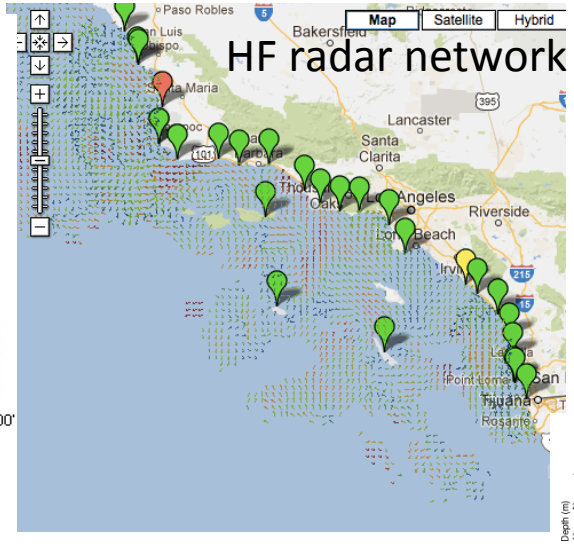
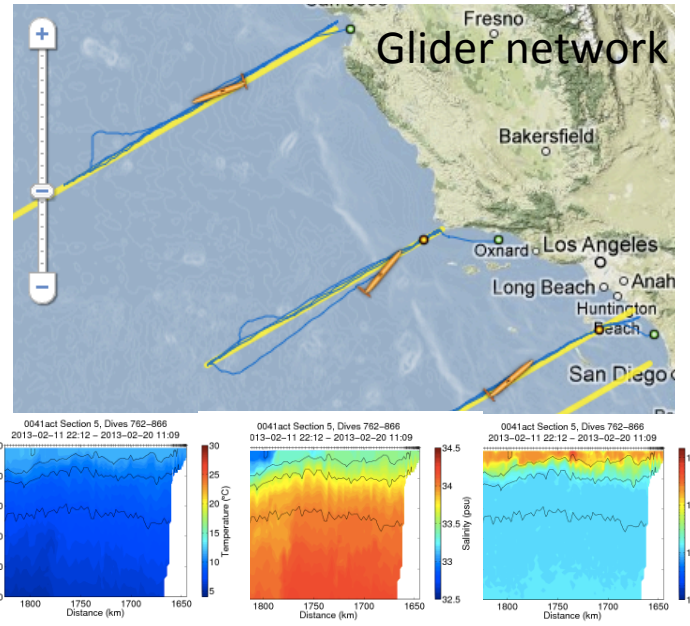
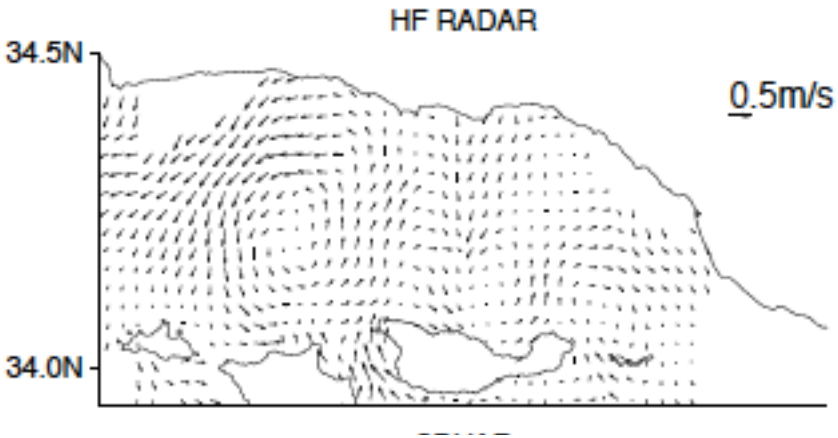
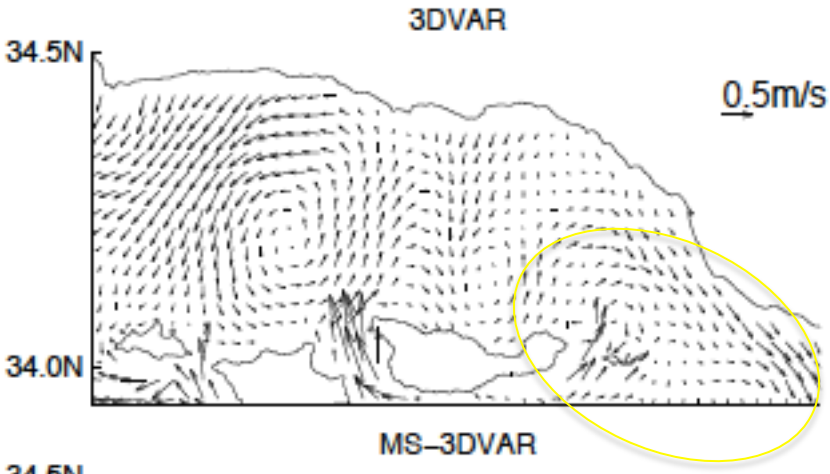


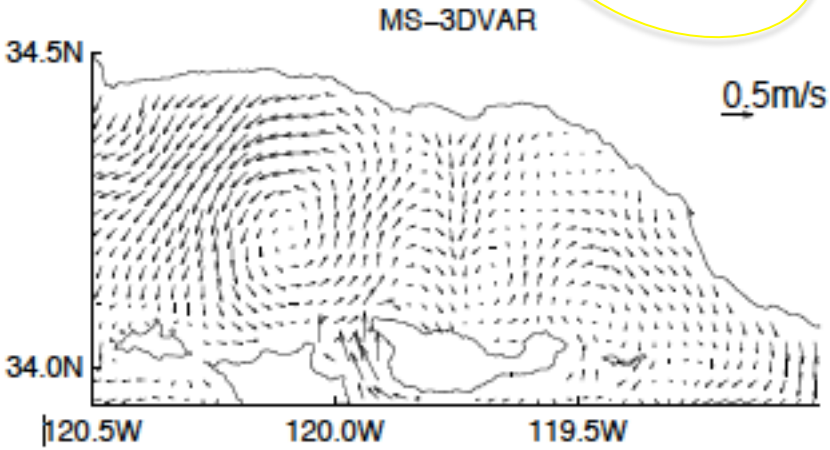


Real Observation Experiments

- HF Radar: 03UTC, August 13, 2008: difference in details
  - 3D-Var
    - Corr (u,v)=(0.53,0.67)
    - RMSD (u,v)=(0.19,0.15) [m/s]
  - MS 3D-Var
    - Corr (u,v)=(0.61,0.72)
    - RMSD (u,v)=(0.17,0.14) [m/s]



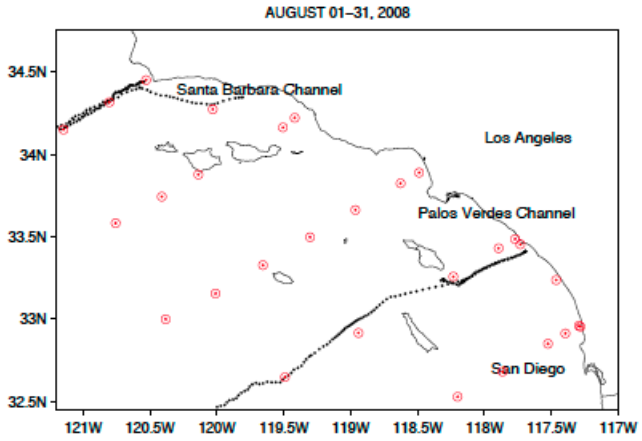




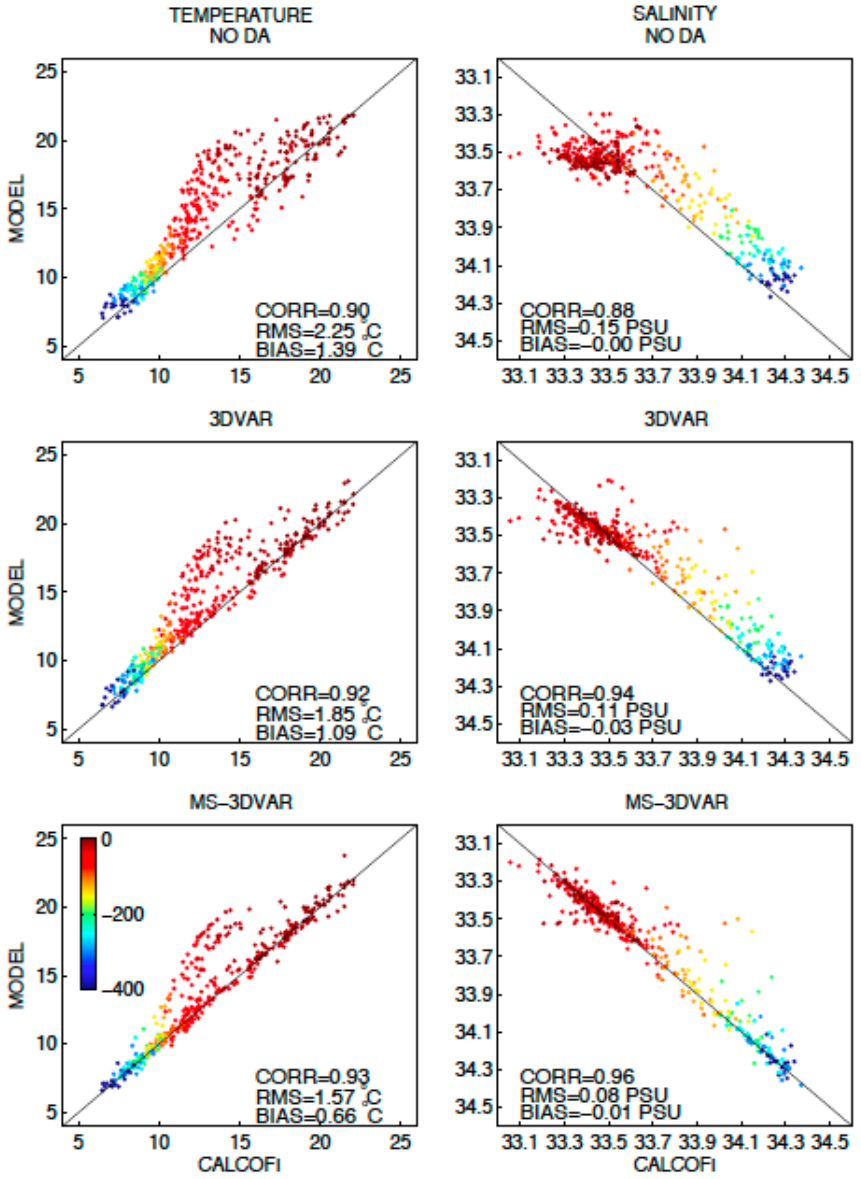
Real Observation: Validation Against Independent Data

- CALCOFI data sets

  (o: not assimilated)



- MS 3D-Var
  - Reduction of bias
  - More observation is needed



Concluding Remarks

- MS 3D-Var works well
  - Two main elements
    - Successive application of localization from Large-scale to smaller scales
    - Separation of observing system network
  - Reduction of bias

- Extension to 4D & LETKF is straightforward:
  - Scale separation $\Delta\mathbf{x}=\Delta\mathbf{x}_L+\Delta\mathbf{x}_S$ with $\Delta\mathbf{x}_L=\mathbf{U}_L\mathbf{w}_L$
    $\Delta\mathbf{x}_S=\mathbf{U}_S\mathbf{w}_S$
  - Scale dependent inflation

- Coastal LETKF itself is challenging: maybe hybrid