Analytical Problems

1. [No need to do this problem]
   Find a function form of $y = \exp(Cx)$ that best fits the data set consisting of 2 data
   a. $(x_1, y_1) = (0, 1/2)$ & $(x_2, y_2) = (0, 1)$
   b. $(x_1, y_1) = (0, A)$ & $(x_2, y_2) = (0, B)$
   c. Verify consistency of your results from (a) and (b)

2. Determine the best approximate solution of the linear system

   \[
   \begin{align*}
   2x + 3y &= 1 \\ 
   x - 4y &= -9 \\ 
   2x - y &= -1
   \end{align*}
   \]

   in the least-square sense.

Computational Problem

3. Generate a data set $\{t(l), b(l)\}$ by
   \[
   b(l) = a_0 + a_1 t(l) + a_2 t(l)^2 + a_3 \sin(t(l))
   \]
   for $l = 1, \ldots, L = 100$, where
   \[
   (a_0, a_1, a_2, a_3) = (2, 3, 0.1, -0.1);
   \]
   \[
   t = \text{linspace}(0, 2, L)';
   \]

4a. Develop a general MATLAB code for the least-square estimation using the
   QR factorization with partial scaled pivoting, following the steps in Exercise 3.
   b. Using the code, compute the coefficient $c_0$ and $c_1$ of the linear function
      \[
      q(t) = c_0 + c_1 t
      \]
      that fits your data generated by Problem 3 in the least square sense.
   c. Plot in one figure
      - all data points
      - linear line that you obtained by the QR decomposition.

5a. Develop a general MATLAB code for the least-square estimation using
   Cholesky factorization following the steps in Exercise 4.
   b. Using the code, compute the coefficient $c_0$ and $c_1$ of the linear function
      \[
      q(t) = c_0 + c_1 t + c_2 t^2
      \]
      that fits your data generated by Problem 3 in the least square sense.
   c. Plot in one figure
      - all data points
      - linear line, quadratic line that you obtained by the Cholesky factorization.