AMSC/CMSC460 Section 2.

Homework Set 6

Due: Tu May 10, 2011. 10:45am
By the end of the class

1. Analytical problem

a. Verify that, when Newton's method is used to compute $\sqrt{a}$ (by solving the equation $x^2=a$), the sequence of iteration is defined by

$$x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n})$$

b. Show that if the sequence $\{x_n\}$ is defined above, then the following holds

$$(x_{n+1})^2 - a = \left( \frac{(x_n)^2 - a}{2x_n} \right)^2$$

2. Analytical problem for Euler Backward method

For a N-dimensional standard ODE

$$\frac{d}{dt} y = f(t, y)$$

starting at $(t_0, y_0)$ with time step $h$

a. Obtain the expression for the global error $y(t_{n+1})-y_{n+1}$ at $t_{n+1}$ given the global error at $y(t_n)-y_n$ at $t_n$

b. Discuss stability condition based on the global error.

3. MATLAB problem

a. Write a MATLAB code to find the root of $f(x)=x^3+2x^2+10x-20$ using the

i) Newton's method

ii) secant method

b. Run the code at most 20 steps for both methods [starting from (x0,x1)=(2,1) for secant method], compare the results, and discuss convergence.

c. MATLAB has a function called "fzero" to find root of the function. Compare your results using your codes with the results obtained by "fzero."

4. MATLAB problem

a. Write general MATLAB code

i) Euler forward method

ii) Euler backward method

b. Consider a 2-dimensional linear ODE

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\omega y \\ \omega x \end{pmatrix}$$

with $\omega=1$ and the initial condition $(x(0),y(0))=(1,0)$.

i) Solve analytically,

ii) Solve numerically with 3 values of h=[0.025, 0.5, 0.1] from time 0 to 5 using Euler forward and backward methods and plot the results in two figures:

Fig. 1: Numerical solutions in 2-D by Euler forward for 3 values of h using different colors for each h, along with corresponding analytical solution in another color. Figure should have either a legend or caption.
Fig. 2: Same as in Fig.1 but by Euler backward.