# A Framework for construction of and computations on Four-Dimensional Aircraft Trajectories 

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#### Abstract

Air traffic control systems perform many functions in order to safely and efficiently move air traffic through the national and international airspace. A key component of these systems is an accurate prediction of where the individual flights will be over time. This project builds a framework to both do that prediction and use the prediction to examine various air traffic control algorithms for both accuracy and efficiency. This framework is design for reuse, such that future research into ATC algorithms can be conducted with simple extensions to the system.


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## 1 Introduction

Global Air Traffic Control (ATC) is distributed among multiple Flight Information Regions (FIRs). Each country is responsible for safely and efficiently handling traffic in their FIRs. Most countries have 1 FIR; due to the volume of air traffic and the complexity of its airspace, the US has 20.

At the core of the FIR's processing is building an estimate of each aircraft's trajectory (a four dimensional description of the flight path, with dimensions of latitude, longitude, altitude and time).

Once these trajectories are constructed, they are used for various purposes, such as conflict detection (which predicts whether two aircraft will be too close to each other, or if a single aircraft will fly into restricted airspace), workload planning for the human controllers separating air traffic, airport arrival and departure planning, etc.

This project will examine different aspects of generating and using these trajectories, including performance aspects of using the trajectories in various applications.

## 2 Project Goal

The project will build a modular system where:

1. Different trajectory generation algorithms can be tested
2. Algorithms using those trajectories can be tested for performance and accuracy. The specific algorithms selected have relevance to the state of the practice today.

This framework can be used to explore new concepts for air traffic management.
A secondary goal is that the project's author becomes familiar with techniques of parallelizing computing algorithms, and becomes more proficient in the use of the Python programming language.

As of December 2016 the initial framework has been created to generate trajectories; spring of 2017 will be dedicated to carrying out a set of experiments using those generated trajectories.

## 3 Approach

The system to be built looks like this:


Figure 1 Framework Design

The trajectory generation algorithm takes three major inputs:

1) Flight intent, consisting of items such as route of flight, departure time, desired airspeed, type of aircraft and desired cruising altitude
Flight Intent can be either synthetic for testing or derived from real-world data (via the publicly available "Aircraft Situation Display to Industry" (ASDI) data stream, or websites such as "FlightAware"[9]).
2) An atmospheric model, listing temperatures, pressures and wind aloft values for the FIR's geographic region. One instance of this is a three-dimensional model, but multiple hour forecasts are available, which adds a fourth dimension. This is publicly available from NOAA, and are distributed in "GRIB2" format[6]. These files typically use a Lambert conical projection, which would be transformed into a grid of fixed sized cells, with fixed sized latitude, longitude and altitude[8].
Initial testing can be done with a "null" model of zero winds.
3) A set of airframe parameters, that indicate how particular aircraft types (such as a Boeing 737) will climb and descend. For this, Eurocontrol's Base of Aircraft Data (BADA) dataset[1] will be used (under the University of Maryland license). There are two versions of this model in use; version 3 (which has a rich set of airframe parameters) and version 4 (which is more detailed but covers fewer airframes currently). Time and licensing arrangements permitting, the differences between version 3 and 4 can be explored.

These experiments are proposed; time constraints may preclude all four from being completed (these are listed in priority order):

1) The current US system uses current-hour weather model for all computations, even though a flight can last several hours. Implement a version that uses future weather forecasts and compare changes in long-duration trajectories
2) Find an optimal wind-aided trajectory. This will find an "optimal" trajectory between two points given non-zero wind patterns. Initially, "optimal" will be define as the trajectory that uses the least fuel; other more complex definitions could be supplied.
3) Parallelize conflict detection algorithms, measure complexity of implementation vs. speed tradeoffs for different parallel techniques
4) Look at the increased accuracy of BADA version 3 vs. version 4 for identical aircraft types, balanced against the increase in complexity and computational load

These are described in slightly more detail in Section 4.7 Proposed Experiments on page 15.

## 4 Scientific Computing Algorithms

The principal algorithm that is implemented is the generation of the 4D trajectory; this requires the solution of the ordinary differential equation to describe the climb and descent segments of the trajectory. Some basic concepts are discussed first.

### 4.1 Coordinate system

Points on the earth's surface are transformed to a Cartesian coordinate ( $x, y, z$ ) on a unit sphere (radius = 1.0). Given the radius of the earth, distances on the surface of the unit sphere are converted to nautical miles (NMI), for things like speed calculations (Knots). Other calculations such as intersection of great circle arcs are simplified when the unit sphere is used.

Since we are concerned with a particular geographic area, an approximation of the earth's radius for that area would be used in practice (perhaps slightly different for each FIR). For this project, the mean radius of the earth will be used.

All external latitude/longitudes are supplied in WGS 84 coordinates[2], which assumes the earth is an ellipsoid with specific major and minor axes (same as is used by the GPS). Translation to the unit sphere is straight forward, and supplied as routines in many programming languages.

Note that the Cartesian coordinate $(\mathrm{x} / \mathrm{y} / \mathrm{z})$ is for the surface of the earth. Altitude above the surface is separately specified (and forms one dimension of each trajectory point). This simplifies the ODE to be solved, since it depends on altitude, not distance from the earth's center.

### 4.2 Speed Definitions

The algorithms use several different "speed" values, as described below.
True Airspeed (TAS) - speed of the aircraft relative to the surrounding air mass (which may be moving with respect to the earth's surface)

Ground speed - speed of the aircraft over the surface of the earth; this is the true airspeed adjusted by the winds moving the air mass. If wind speed and TAS are given as vectors, $\mathrm{V}_{G S}=\mathrm{V}_{\mathrm{TAS}}+\mathrm{V}_{\text {wind }}$.

Calibrated airspeed (CAS) - airspeed as known to the aircraft's instrumentation; this is based on a "calibration" that takes into account any known bias in the instruments. For a constant CAS, TAS will vary depending on altitude

Mach - speed of sound; important here because aircraft plan to hold CAS and Mach constant; there is a crossover altitude at above which planning is done based on Mach speed, CAS and Mach will be equal at that altitude.

### 4.3 Trajectory Generation Basics - latitude and longitude dimension

As noted above, the trajectory generator takes flight intent as an input. This is supplied by the airline or pilot, and consists of:

- The path (over the surface of the earth) to be followed
- The desired cruise altitude
- The desired cruise speed (true airspeed or Mach; in practice only military flights specify this in Mach)
- The type of airplane (e.g. Boeing 737)
- Departure time

As an example, consider Southwest flight 3156 from Baltimore (airport code BWI) to Orlando (MCO):

- altitude 40,000 feet
- speed 452 knots
- Boeing 737-300
- Depart at 10:35 am

The creation of the actual path from departure airport to destination airport is beyond the scope of this project; we start with the expanded set of points as an input. This input defines a path on the earth's surface, which starts at the departure airport, ends at the destination airport, and has intermediate waypoints. The waypoints are connected by great circle arcs. In order to more accurately follow the path of the flight, turn waypoints can be replaced with a number of smaller segments that more closely approximate the turn to be taken. Whether this is done depends on the accuracy needed for other work. Figure 2 shows an example path from Baltimore to Orlando.


Figure 2 Horizontal path for sample flight

### 4.4 Trajectory generation basics - altitude and time

In order to turn the two-dimensional waypoints into points on a four-dimensional trajectory, altitude and time are calculated for each point. This results in a vertical profile that looks something like Figure 3.


Figure 3 Vertical profile of sample flight
Each trajectory point is called a cusp, and the segments that connect cusps have a constant rate of vertical velocity (climb or descent) and a constant rate of acceleration over the segment. This, along with the starting ground speed for each segment, allows us to predict where a flight will be at any point in time.

This final representation of the trajectory has additional points added between the waypoints supplied as inputs, in order to assure that the above definition of constant acceleration/vertical velocity holds. For example, it is necessary to accelerate to the desired climb speed; when that acceleration is complete a cusp is added to the trajectory.

### 4.5 Rate of Climb/Descent

Calculation of the rate of climb (or descent) is described in detail in [1], and is summarized here.
The basis of this rate is this conservation of energy equation:

$$
\begin{equation*}
(\mathrm{Thr}-\mathrm{D}) * \mathrm{~V}_{\mathrm{TAS}}=\mathrm{mg}_{0} \frac{\mathrm{dh}}{\mathrm{dt}}+\mathrm{mV}_{\mathrm{TAS}} \frac{\mathrm{dV}_{\mathrm{TAS}}}{\mathrm{dt}} \tag{4.5-1}
\end{equation*}
$$

Where

| Thr | Thrust supplied by the aircraft <br> engines | $\mathbf{m}$ (mass) | Mass of the aircraft, including <br> fuel, passangers and baggage. |
| :--- | :--- | :--- | :--- |
| D (Drag)Drag from movement through the decrease over the life <br> atmosphere | $\mathbf{g}_{0}$ | of the flight <br> Gravitational acceleration |  |
| $\mathbf{V}_{\text {TAS }}$ | Velocity in true airspeed | h | Geodetic altitude |

Combine this equation with the assumption that, during climbs and descents, pilots hold speed (CAS or Mach) and throttle position constant, the above can be re-written as

$$
\begin{equation*}
\frac{\mathrm{dh}}{\mathrm{dt}}=\frac{(\mathrm{Thr}-\mathrm{D}) * \mathrm{~V}_{\mathrm{TAS}}}{\mathrm{mg}_{0}}\left[1+\left(\frac{\mathrm{V}_{\mathrm{TAS}}}{\mathrm{~g}_{0}}\right)\left(\frac{\mathrm{d} \mathrm{~V}_{\mathrm{TAS}}}{d h}\right)\right]^{-1} \tag{4.5-2}
\end{equation*}
$$

The last term can be replaced by an "energy share factor" [3]; the ratio of energy allocated to climb vs. acceleration for a constant velocity; a function of Mach:

$$
\begin{equation*}
f\{M\}=\left[1+\left(\frac{\mathrm{V}_{\mathrm{TAS}}}{\mathrm{~g}_{0}}\right)\left(\frac{\mathrm{dV}_{\mathrm{TAS}}}{d h}\right)\right]^{-1} \tag{4.5-3}
\end{equation*}
$$

Hence, the rate of climb or descent (which is typically expressed in pressure altitude $H_{p}$, rather than geodetic altitude h), is

$$
\begin{equation*}
R O C D=\frac{d H_{p}}{d t}=\frac{T-\Delta T}{T}\left[\frac{(\mathrm{Thr}-\mathrm{D}) * \mathrm{~V}_{\mathrm{TAS}}}{\mathrm{mg}_{0}}\right] f\{M\} \tag{4.5-4}
\end{equation*}
$$

Where $T$ is temperature at altitude, and $\Delta T$ is the temperature differential at that altitude from the International Standard Atmosphere (this is given by the atmospheric model).

### 4.5.1 Energy Share Factor

The energy share factor $f\{M\}$ is a function of Mach ( $M$ ) and varies with altitude, and whether we are accelerating or decelerating in climb or descent. As an example (to convey the complexity of the equation), below the crossover altitude when not accelerating, it is defined as:

$$
\begin{equation*}
f\{M\}=\left\{1+\frac{\kappa R \beta_{T,<}}{2 g_{0}} M^{2} \frac{T-\Delta T}{T}+\left(1+\frac{\kappa-1}{2} M^{2}\right)^{\frac{-1}{\kappa-1}}\left\{\left(1+\frac{\kappa-1}{2} M^{2}\right)^{\frac{\kappa}{\kappa-1}}\right\}-1\right\}^{-1} \tag{4.5-5}
\end{equation*}
$$

Where
$\boldsymbol{\kappa} \quad$ Adiabatic index of air $=1.4$
R Real gas constant for air $=287.05287$
$\boldsymbol{\beta}_{\boldsymbol{T},<} \quad$ ISA temperature gradient below the tropopause - -0.0065

The Mach number is based on the current speed at the current altitude, which may be a true airspeed. If so, that TAS is converted to a Mach value by dividing the $\mathrm{V}_{\text {TAS }}$ by the speed of sound at the given altitude (the value for T is obtained from the weather model at the current altitude):

$$
\begin{equation*}
M=\frac{V_{T A S}}{\sqrt{\kappa R T}} \tag{4.5-6}
\end{equation*}
$$

The full definition of the energy share factor can be found in [1].

### 4.5.2 Thrust

Thrust depends on the engine type (Jet, Turboprop or Propeller), pressure altitude, airspeed (in some cases) and temperature differential.

For example, for a jet engine, the maximum climb thrust would be:

$$
\begin{equation*}
\left(T h r_{\max \operatorname{climb}}\right)_{I S A}=C_{T C, 1} *\left(1-\frac{H_{p}}{C_{T C, 2}}+C_{T C, 3} * H_{p}^{2}\right) \tag{4.5-7}
\end{equation*}
$$

Where the coefficients $C_{x}$ are given for the airframe type in the BADA dataset. Note that in this case (of a jet engine) there is no term for airspeed.

### 4.5.3 Drag

Drag depends on the Drag Coefficient, which in turn depends on the lift coefficient.
Lift coefficient:

$$
\begin{equation*}
C_{L}=\frac{2 * m * g_{0}}{\rho * V_{T A S}^{2} * S * \cos \varnothing} \tag{4.5-8}
\end{equation*}
$$

Drag Coefficient:

$$
\begin{equation*}
C_{D}=C_{D 0, C R}+C_{D 2, C R} *\left(C_{L}\right)^{2} \tag{4.5-9}
\end{equation*}
$$

And finally drag force:

$$
\begin{equation*}
D=\frac{C_{D} * \rho * V_{T A S}^{2} * S}{2} \tag{4.5-10}
\end{equation*}
$$

S Wing surface area
$\rho \quad$ Air density
$\emptyset \quad$ Bank angle (zero, unless in a turn)

### 4.5.4 Weather Model

The weather model supplies some key data for the above equations. Specifically, the following information is used from the NOAA-supplied data:

1) North wind component
2) East wind component
3) Temperature (from which the $\Delta T$ can be calculated)
4) Pressure

### 4.6 Trajectory Generation Summary

### 4.6.1 Flight Intent Source

The FlightAware web site (www.flightaware.com) provides intent (airspeed, cruise altitude, route of flight, departure time) for all commercial flights in the US. Once a pair of endpoint airports is selected, that site provides the fixes in the route for a variety of flights between those cities. This data is screenscraped (using a developed Python program) to produce an intent file that can be used in various experiments.

Flights of various durations and directions can therefore be added to the inventory of test data.

### 4.6.2 Airframe Parameters

Eurocontrol's BADA database is used (by extending the University's existing license for use on this project); version 3.13 is the latest version as of fall 2016; it is used for this effort.

### 4.6.3 Atmospheric Model

GRIB2 files, with atmospheric levels, are provided hourly by NOAA. Selected days data is captured and converted to a form usable by the program. That conversion follows this process:

1) The data from NOAA is provided in various resolutions; for this effort the 40 km grid is used, with the data covering the continental US.
2) That data is processed by NOAA's "degrib" tool [10] to produce ASCII files (in comma-separatedvalue format, CSV) containing:
a. Grid Coordinates (x/y)
b. Latitude and Longitude (in geodetic coodinates) of that grid point
c. Value for the parameter; the temperature, $U$ wind value, $V$ wind value and pressure values are used here
3) Data in the CSV file is in the Lambert Conical Projection, with a secant line of 25 degrees north latitude. When building a trajectory, fast access to weather data in a spherical (conformal) grid is needed. Hence the data is transformed to a lat/long grid in equally-spaced conformal latitude and latitude, in equally spaced altitude bands of 1000 feet. This data is then written to disk using the "numpy" Python package (which stores dense arrays of floating point data). During trajectory generation, the particular set of weather data and these files are read for use during processing. Numpy is used in various places in this project, but (at this point) only for its efficient in memory representation of large arrays, and the load and store operations on those arrays.

The transformation from Lambert to Spherical requires that a spherical grid be constructed that lies within the Lambert data. The values of each parameter at the spherical grid points is an interpolated value from the Lambert grid. In the horizontal direction, a bi-linear interpolation is used from the four surrounding Lambert grid points; these horizontal points (which are at pressure levels 1000 mb through 100 mb ) are then linearly interpolated in the vertical direction to give values at evenly spaced 1000 foot altitude bands. This is the same technique as is detailed in "ERAM Flight Data Processing (FDP) and Weather Data Processing (WDP) Algorithms" [8].

The grids are shown in Figure 4. This figure is shown with latitude distances unchanging with longitude (that is, no special projection). The blue Lambert grid appears to expand towards the north, since in fact the longitudinal spacing will decrease as we go north. The red spherical grid that is created by the program is designed to fit completely in inside the Lambert grid, so that only interpolation is needed to find point values. The green box shows the extent of the lower 48 states; the part that is outside the red grid to the east is the eastern-most tip of Maine. If this setup modeled a flight in that remote part of Maine, no wind or temperature information would be available.


Figure 4 Lambert vs. Spherical grids

### 4.6.4 Trajectory Generation

Using the above definitions, the ordinary differential equation given for ROCD (Equation 4.5-4) is solved using a Runge-Kutta technique (Python provides some existing code in this area, scipy.integrate.ode provides Runge-Kutta solutions for order 4 and 8 with configurable step size). This describes the climb or descent phases; the cruise phase (moving at constant true airspeed) is simply following the route of flight from one waypoint to another. Note that the time associated with each cusp is calculated from the ground speed and ground speed acceleration applied to get from one cusp to the other. Ground speed is derived from the true airspeed by applying the wind observed at the altitude of the cusp.

While the descent phase also involves solving the ODE for ROCD, there is one additional complication; that equation requires mass to be known at each cusp. In this case, we know where we want to end (the destination airport), but the point at which to start the descent is not precisely known. An iterative technique is used:

1) First, make a gross approximation of the top of descent point.
2) Solve the ODE, descending to the destination airport's altitude.
3) If we are within a small epsilon of the airport, the iterations are done. The choice of epsilon affects how many iterations are necessary; in this case, an epsilon of approximately 1000 feet (shorter than commercial airport runways) is appropriate.
4) Otherwise, move the top of descent point forward or backward along the route defined by the waypoints, in order to get closer to the destination airport, and repeat starting at step 2.

The movement of the top of descent point can be summarized as follows:

1) Let $T O D_{k}$ represent the top of descent point for iteration " $k$ ". This is measured in distance along the route from the departure airport. This is termed the "along route distance", or ARD.
2) Let AP represent the ARD of the destination airport; this is the known distance at the end of the trajectory
3) On any iteration $k$, the ARD when the trajectory is modeled to reach the destination airport is denoted as $\mathrm{D}_{\mathrm{k}}$.
4) Let function $E$, at any iteration $k$, represent the difference between the actual outcome ( $D_{k}$ ) and the desired outcome (AP). When this function is zero, we are done. So $E_{k}=\left(D_{k}-A P\right)$.
5) The algorithm becomes:
a. Take an initial guess at $\mathrm{TOD}_{0}$; this will be based on some standard descent rate.
b. Calculate $\mathrm{D}_{0}$.
c. Set TOD $=$ TOD $_{0}-E_{0}$
d. Compute $E_{k}$ (initially for $k=1$ )
e. If $E_{k}$ is not sufficiently small, set

$$
\begin{equation*}
T O D_{k+1}=T O D_{k}-\frac{E_{k}\left(T O D_{k}-T O D_{k-1}\right)}{E_{k}-E_{k-1}} \tag{4.6-1}
\end{equation*}
$$

f. and repeat steps (d) and (e)

In practice, this algorithm converges in 2 or 3 iterations. With complex weather patterns, it may take more, as these may introduce discontinuities in the relationship between TOD and total length of descent. The program also limits the number of iterations to some large number (such as 50 ) in order to guard against endless loops (if these limits are reached; the trajectory generation process is abandoned).

This yields the four-dimensional trajectory, which can be used in the proposed experiments listed below.

### 4.7 Proposed Experiments

### 4.7.1 Trajectory Generation

Prior to any experiment using a trajectory, the trajectories have to be generated and verified.
In general, the climb/cruise/descent phase is best depicted as a plot of altitude against either elapsed time from take, or distance from takeoff. An example of Altitude vs. Distance is shown in Figure 5.

## Python Generated Trajectory

Boeing 735 - Cruise Alt 35,000


Figure 5 Typical Altitude vs. Distance Plot
Each point on this plot is the end of a trajectory segment; each point has a latitude, longitude, altitude and time, and each segment has a rate of climb or descent and ground speed acceleration.

### 4.7.2 Use of Weather Forecast Models

A weather model (temperatures, pressures and wind speed) is essential for trajectory generating. The current state of the practice uses the current weather model for the entire trajectory generation process, even though the trajectory itself may span many hours. NOAA provides not only the current hour weather model, but predictions for future hours as well.

This experiment will modify the trajectory generation process to use those forecast models, interpolating between hours to get an approximation of the variables at the cusp times in the trajectory.

This increases the complexity (slightly) and the storage needed (more than slightly) of the algorithm, and the question to be answered is whether this substantially changes the produced trajectory, especially the duration of the trajectory.

### 4.7.3 Wind Optimal Trajectories

The trajectories as described above follow a known path along the surface of the earth between waypoints. By following these paths, separating aircraft becomes somewhat simpler; especially to human air traffic controller who is used to aircraft following known paths.

By using the rigid path, however, the time in the air may be longer than necessary, especially if there are favorable wind conditions on other paths. This experiment will use Particle Swarm Optimization (PSO) techniques [4][5] to find an optimal path from fixed end points (the departure and destination airports are fixed, obviously). In this case, "optimal" can be defined as a function by an airline operator; the simplest choices are total flight time or fuel burned.

Note that there have been debates in the air traffic community about so-called "free flight" (letting aircraft fly where they want to, rather than where the air traffic control system dictates). This experiment will make no assumption on the future plans of the FAA to encourage or discourage free flight; rather the emphasis is applying PSO to this problem space.

PSO is used to find paths between two points ( $A$ and $B$ in this discussion). Since the initial departure path from an airport is determined by available runways and wind conditions, point A will start at what is called the "Meter Fix"; a point after departing the airport. Similarly, point B will be the meter fix on approach to the destination airport.

PSO examines several paths from A to B. A fuel usage value is calculated for each path, and PSO iterates until it appears that the improvements in additional paths is minimal. PSO doesn't guarantee that a minimum value is found; rather the goal is to find an acceptable solution in a relatively small amount of time.

The algorithm contains a bit of randomness (simulating the somewhat random behavior of a swarm), but learns from each iteration as to where the next iteration should concentrate. The basic operation of the PSO algorithm is shown in Figure 6.

One iteration of the algorithm will find many paths. Any given path is determined by a set of points. Figure 6 shows 5 possible paths starting at point A. The first iteration always proceeds this way; the number of paths and the range covered by those paths will be determined by experimentation. One segment in one path is a straight line of a given distance, ending at point $P$ in the figure. The next segment will be of the same length; the direction of the segment is constructed based on three values:

1) The course coming into point $P$. This is called $\Theta_{i}$ (for inertia). There is some randomness introduced into this value, within a range $+/-$ around the incoming course.
2) The course with the optimal tail wind at point $P$. This is called $\Theta_{w}$ (for wind). Recall that the wind values are in a three dimensional grid, so altitude matters when determing the wind. For the departure segments from point A , altitude is determined from the ROCD calculations given in section 4.5. For approach segments to point B , this is more difficult as the general algorithm includes an iterative approach to find the top of descent point (section 4.6.4). As an approximation, a standard descent profile is built in the absence of winds, going from the cruise altitude to the altitude of point $B$, and once a new point $P$ is within that (direct) distance of $B$, the altitude is retrieved from that standard profile at the distance ( P -> B) from the end point.
3) The course from $P$ to the endpoint $B$. This is called $\Theta_{e}$ (for end point).


Figure 6 PSO Algorithm
These three courses are combined with a set of weights:

$$
\begin{equation*}
\theta=W_{i} \theta_{i}+W_{e} \theta_{e}+W_{w} \theta_{w} \tag{4.7.3-1}
\end{equation*}
$$

These weights sum to 1.0. In order to converge on B, the weights are calculated as follows.
$W_{e}$ is calculated first, according to how close we are to the endpoint:

$$
\begin{equation*}
W_{e}=\left(1-\frac{\operatorname{dist}(P, B)}{\operatorname{dist}(A, B)}\right) \tag{4.7.3-2}
\end{equation*}
$$

Additionally, once $W_{e}$ is greater than a threshold (say, 0.95 ), it will be set to 1.0 . This ensures we eventually get directly to point B.

Once $W_{e}$ is known, the remaining weight (1.0- $W_{e}$ ) is divided between $W_{i}$ and $W_{w}$ based on predetermined percentages; say $70 \%$ for wind and $30 \%$ for inertia. This ratio is subject to experimentation.

When the ultimate course $\theta$ is known, a new point $P$ can be calculated as the next step on the path.
Each path in the iteration is calculated in this fashion. This will result in a set of zig-zag lines. These lines are smoothed to represent flyable paths, and the fuel consumption for each path is calculated.

That concludes one iteration. The lowest-fuel consumption path is chosen as the "best" path for that iteration.

Any iterations after the first differ only in the starting courses. Since a previous best path is known, the starting courses from point $A$ are now chosen to cluster around that previous best path.

Iterations proceed until the delta fuel usage for the best path between iterations is less than some selectable value epsilon.

### 4.7.4 Parallelizing Conflict Detection

Once a flight information region has a set of trajectories (flights that are currently airborne or are scheduled to be airborne within some time horizon), the ATC systems must predict if any two aircraft will come too close to one another, or if any aircraft will enter restricted airspace.

The aircraft-to-aircraft case is interesting in that it is a compute-intensive activity that might lend itself to parallelization. At its core, this process must compare any new or changed trajectory against all other trajectories in the FIR's set of flights. Since each trajectory consists of a set of segments, this entails many segment-to-segment comparisons. A typical FIR may have 200 or so flights in its working set, each of those trajectories may have 200 or so segments. A completely brute force approach would then, for a single new or modified flight, do 200 (segments in new trajectory) $\times 200 \times 200$ (segments in existing trajectories), or 8,000,000 comparisons.

In practice [7], this is implemented as a sequential process where the vast number of comparisons is reduced by a set of filters (the filters weed out cases that could not possibly be in conflict). The first two filters listed below are on the trajectories as a whole; the remaining are for each segment on the two trajectories being checked.

These filters are:

1) Trajectory level coarse time filter - if one trajectory ends before the other starts, they cannot be in conflict
2) Trajectory level box filter - this compares the "box" constructed in three dimensions (latitude, longitude and altitude) for each of the two trajectories being compared, to see if there is overlap at that level
3) Segment level vertical coarse filter - this checks to see if the two segments are within the same altitude band
4) Segment level horizontal coarse filter - this checks the minimum approach distance in a horizontal direction
5) Segment level horizontal middle filter - again checks for closest approach, but now considers where the aircraft might be within the segment
6) Segment level vertical middle filter - again considers altitude, but only on the portion of the segments that come within the horizontal separation distances
7) Segment level horizontal fine filter - checks for closest approach, but only the regions that have passed the previous filters

A true conflict is one that passes all filters. By parallelizing the ultimate checks done by the last filter, it may be possible to avoid some of the intermediate filters to achieve performance gains.

### 4.7.5 BADA Version 4 Complexity vs. Accuracy

Eurocontrol has published a version 4 of the BADA model. This includes more complex equations, requiring more parameters per airframe type, in order to achieve a higher degree of accuracy. If we can get access to the version 4 model (it is a separate licensing request to Eurocontrol), this experiment would compare the increase in complexity and run time vs. the accuracy gained by using version 4 vs. version 3.

## 5 Implementation

The implement is done using the Python language. It offers the following advantages:

- Portable to many operating systems.
- Has support for parallel processing, including packages that interface to GPUs.
- Has the concepts of exception handling, classes, objects, modules, and packages.
- Has a wealth of other packages for scientific computing.

In spite of not being strongly typed or compiled before execution, this makes Python attractive for this effort.

For an IDE, the PyDev Eclipse plug-in [11] is used. Git [12] is used for configuration management. Python offers some limited documentation support in the language, but a separate design document is among the deliverables; this will discuss major code structure and design decisions.

The development platform is a personal computer (desktop). Tests will also be run on this configuration. For the parallel processing experimentation, it will be necessary to obtain a high end graphics card (for use in a desktop computer); the Nvidia GEFORCE 960 graphics card is chosen for this purpose.

If multiple general purpose CPUs networked together are needed, inexpensive Raspberry Pi computers can be used.

## 6 Validation Methods

Eurocontrol conveniently provides a tool that uses their version 3 equations to produce a 4D trajectory. This can be used to verify the basic trajectory generation engine. This tool does not assume any weather model; adding a non-zero-wind model can be verified through spreadsheet analysis of the produced trajectories.

The experiments that use a trajectory will be validated through careful construction of test cases (using alternative computations to ensure the necessary conditions are created), and then running those test cases to be sure the desired result is obtained.

### 6.1 Trajectory Generation

The verification occurs on two levels, and takes advantage of the fact that a known, proven implementation exists (from real air traffic control (ATC) systems). First, the low level equations for thrust, drag, etc., are verified against existing implementations of those equations (same inputs should produce the same outputs). Second, the overall trajectory that was generated is compared against a similar result from an ATC system simulator.

This overall comparison for a descent segment is shown in Figure 7.


Figure 7 Trajectory Generation Comparison
There are two places where there are noticeable differences:

1) At the top of descent, both cases shown a period of acceleration, during which the desired descent speed is attained (this speed is different than the cruise speed of the aircraft, requiring this period of acceleration). During this period, the rate of descent will be smaller than normal, because energy is being used to change the speed. The Python generated trajectory ends this period sooner than the ATC version. A detailed analysis was undertaken for this; it revealed a bug in the ATC code (wrong value for acceleration was being used).
2) The Python generated trajectory ends sooner than the ATC trajectory. This is simply a function of the flight intent; the flight chosen for generation in Python was destined for an airport at 285 feet above sea level; the ATC flight was going to an airport at 14 feet above sea level. Hence the Python trajectory ends sooner.

The above comparison is done without any wind (in order to have a valid comparison, wind data must be equal, and there is no way to use the same wind conditions in both cases). Another test is needed to show that the wind speeds are applied properly. For this, a "windy" day was chosen, where there was a significant head wind in the Python's flight direction. This is expected to slow down the aircraft.


Figure 8 Effect of Wind on Trajectories

In both plots, the blue line is the trajectory with the head wind applied; the red line shows the same flight without wind. In an altitude vs. time plot, the climb and descent portions are identical (since the calculation of rate of climb/descent (Equation 4-5.4) does not depend on ground speed, only true airspeed. However, in this case, the cruise segment is elongated by more than 6 minutes due to the head wind. The same information presented as an altitude vs. distance plot shows the flight is at cruise for a longer distance, as the ground speed at cruise is reduced by the head wind.

## 7 Test Problems for Verification

### 7.1 Use of Weather Forecast Models

Several days of real weather models will be obtained by NOAA, especially ones where there are significant changes in wind speeds. Starting with the same flight intent input, several trajectories can be generated using both the current forecast only and the future forecasts. These trajectories will be chosen to traverse the FIR in different directions, in order to experience head and tail winds.

The total flight time of each flight will be compared.

### 7.2 Wind Optimal Trajectories

Using both the weather models used above and synthetic models designed to have greatly varying wind speeds in different region, the Particle Swarm Optimization techniques will be used on a single flight.

### 7.3 Parallelizing Conflict Detection

An entire set of flights, as typically would be found in a FIR, is not needed for this experiment. Rather, we just need pairs of flights that show various degrees of "closeness" to being in conflict with one another. These will be run through a parallel algorithm, in order to validate the proof of concept (that is, what is the proper way to parallelize this problem).

A full comparison would need to compare parallel techniques vs. state-of-the-practice techniques. I don't anticipate enough time in this course to complete that comparison.

### 7.4 BADA 3 vs 4

The details of the BADA version 4 implementation are not known in detail (they won't be known until we obtain a license for them). Hence the particular test cases where version 4 should be an improvement aren't known at this time.

Once a license is obtained, these tests can be defined.

## 8 Results

For these main experiments:

1. Using future weather forecasts as opposed to only the current conditions should yield improvements in flight duration times at little computation or complexity expense; memory used would increase. All these values can be quantified in the results.
2. Optimal wind-aided trajectories should find "less expensive" trajectories than nominal case. The reduction in expense can be quantified. In addition, examining some real NOAA data can show how likely it is that alternate, less expensive routes can be found.
3. Parallel solutions for the conflict detection algorithms should reduce computation time, perhaps meaning that simpler algorithms can be used in place of the existing state of the practice
4. BADA version 3 vs version 4, it is expected that version 4 offers greater accuracy, especially in low altitude situations, but may not be worth the extra complexity and computation cost. Note: this requires an additional Eurocontrol license (to get the additional airframe data), and will be done as time allows.

## 9 Conclusion

The creation of a framework to produce trajectories, given the parameters found in a BADA dataset, can be used for many experiments of interest to ATC. This project establishes that framework and conducts a few such experiments. These experiments are designed to provide insight into situations confronting ATC today, and the framework can continue to be used for future work.

## Appendix A. Timeline

## Table 1 Proposed Schedule

| Timeframe | Progress Achieved |
| :--- | :--- |
| Thanksgiving | Implement trajectory generation using BADA version 3-completed <br> Detail the particle swarm optimization algorithm to be used - completed |
| December | Implement and measure the use of forecasted weather |


| January | Initial PSO algorithm for wind-aided trajectories |
| :--- | :--- |
| February | Initial parallel conflict detection algorithm |
| March | Additional parallel algorithm tests, tuning PSO algorithm |
| May | Final algorithms and analysis, produce final report |
| If time allows | Implement BADA 4, compare BADA 3 vs 4 |

## Appendix B. Milestones

The milestones roughly follow the schedule presented, including these items:

1) Basic Trajectory Generation algorithm implemented and tested
2) Initial version of each algorithm implemented
3) Final version of each algorithm implemented and tested
4) Final report produced

## Appendix C. Deliverables

1) Python Source Code
2) Design documentation, including interface definitions
3) Results
4) Class presentations and reports

## Appendix D. Bibliography

| 1. | Eurocontrol Base of Aircraft Data (BADA), http://www.eurocontrol.int/services/bada |
| :--- | :--- |
| 2. | "World Geodetic System - 1984", <br> www.unoosa.org/pdf/icg/2012/template/WGS_84.pdf |
| 3. | Aircraft Modelling Standards for Future ATC Systems; EUROCONTROL Division E1, <br> Document No. 872003 |
| 4. | Kennedy, J. and Eberhart, R. C. Particle swarm optimization. Proc. IEEE int'I conf. on <br> neural networks Vol. IV, pp. 1942-1948. IEEE service center, Piscataway, NJ, 1995. |
| 5. | http://www.swarmintelligence.org/tutorials.php |
| 6. | GRid in Binary (GRIB), the World Meteorological Organization (WMO) Standard for <br> Gridded Data, http://dao.gsfc.nasa.gov/data stuff/formatPages/GRIB.html |
| 7. | "ERAM Conflict Management, Off-Line Problem Determination, and Utility <br> Algorithms", FAA document FAA-ERAM-2008-0423 |
| 8. | "ERAM Flight Data Processing (FDP) and Weather Data Processing (WDP) Algorithms", <br> FAA document FAA-ERAM-2006-0045 |
| 9. | "Filghtaware - Flight Tracking/Flight Status", flightaware.com |
| 10. | National Digital Forecast Database GRID2 Decoder, <br> https://www.weather.gov/mdl/degrib home |
| 11. | Python IDE for Eclipse, http://www.pydev.org |
| 12. | Git fast version control, https://git-scm.com |

## Appendix E. List of Abbreviations

| ASDI | Aircraft Display for Industry |
| :--- | :--- |
| ATC | Air Traffic Control |
| BADA | Base of Aircraft Data |
| CAS | Calibrated Air Speed |
| FAA | Federal Aviation Administration |
| FIR | Flight Information Region |
| GPS | Global Positioning System |
| GRIB | Grid in Binary |
| ISA | International Standard Atmosphere |
| NMI | Nautical Miles |
| NOAA | National Oceanic and Atmospheric Administration |
| PSO | Particle Swarm Optimization |
| ROCD | Rate of Climb or Descent |
| TAS | True Air Speed |
| WGS | World Geodetic System |

