Exercise 2. Construction of Static Background Covariance Matrix

Expected for Upcoming Projects: No Due Date

1. Using a data of limited size \( M \) generated from the Gaussian pdf \((x_0, P_0)\),
   - reconstruct the parameters of the Gaussian pdf \((x_R, P_R)\) and
   - study behavior of \((x_R, P_R)\) wrt \((x_0, P_0)\) as \( M \) increases
     
     for i) case by case & ii) statistical sense:
     
     a. 1D Gaussian: \((x_0, P_0) = (0, 1)\)
     
     b. 2D Gaussian:
        
        \[ x_0 = (0, 0)^T \quad \text{&} \quad P_0 = U_0 S_0 U_0^T \]
        
        with \((\sigma_{01}^2, \sigma_{02}^2) = (2^2, 1^2)\)
        
        (i) zero cross-correlation \( \theta_0 = 0 \)
        
        (ii) non-zero cross-correlation \( \theta_0 = 30^\circ \)
        
        \[
        S_0 = \begin{pmatrix}
        \sigma_{01}^2 & 0 \\
        0 & \sigma_{02}^2
        \end{pmatrix}, \quad U_0 = \begin{pmatrix}
        \cos \theta_0 & \sin \theta_0 \\
        -\sin \theta_0 & \cos \theta_0
        \end{pmatrix}
        \]

2. In data assimilation, background covariance \( B \) (of \( P^b \)) may be constructed from data obtained by the model integration. Read the paper and understand how the “NMC method” works (so that you will be able to improvise & implement)