### Project III. Extended Kalman Filter

- **Dates**
  - 10min presentation: 2013.03.28 & 04.02 in-class
    - Order to be announced
  - Report: 2013.04.03 5pm by email

- **Objectives:** Implementation of Extended Kalman Filter based on
  - Data Assimilation Framework developed in Project 1
  - Dynamic extension of OI in Project 2
    - Understanding of dynamic propagation of uncertainty using tangent linear model, and accuracy
    - Schemes to improve the performance: Inflation of the error covariance
    - Understanding of nonlinearity
  - Enhancement of diagnostic tools
Step 1. Forecast \((x^b_k, P^b_k)\)
- Obtained by integrating
  \[
  \frac{d}{dt} x = f(x, t), \quad \frac{d}{dt} P = FP + (FP)^T
  \]
  starting from \((x^a_{k-1}, P^a_{k-1})\) over \([t_k, t_{k+1}]\).

Step 1. Model Forecast
- \((x^b_k, P^b_k)\)
  - dyn. forecast from \((x^a_{k-1}, P^a_{k-1})\)

Observation
- Measurement: \(y^o_k\)
  \[
  y^o_k = h_k(x^t_k) + \epsilon^o_k : y \in \mathbb{R}^L, \epsilon^o_k \sim N(0, R^o_k)
  \]

Step 2. Analysis
- \(x^a_k = x^b_k + K_k \left( y^o_k - H_k x^b_k \right)\)
- \(P^a_k = (I - K_k H_k) P^b_k\)
- \(K_k = P^b_k H_k^T \left( H_k P^b_k H_k^T + R^o_k \right)^{-1}\)

Application of inflation to \(P^b\)
- Approach I in Step 1
  \[
  \frac{d}{dt} P = FP + (FP)^T + Q^b
  \]
- Approach II in Step 2
  a) \(P^b \Rightarrow P^b + Q^b T_w\) : Additive
  b) \(P^b \Rightarrow \rho^b P^b\) \(\rho > 1\) : Multiplicative

\(\epsilon^o_k \sim N(0, R^o_k)\)

\(d/dt x^t_k = f(x^t_k, t)\)

\(d/dt P = FP + (FP)^T + Q^b\)
Project III. Extended Kalman Filter

- **Specifics**
  - Implementation of Extended Kalman Filter (EKF)
    - Step 1. Simultaneous forecast of $x^b(t_k)$ and $P^b(t_k)$ along $x^b(t)$-evolution
    - Step 2. Analysis of $x^a(t_k)$ and $P^a(t_k)$
  - Validation
    - Step 1
      - TLM (e.g., Exercise 4)
      - $P^b(t_k)$: check positive definiteness & symmetry
    - Step 2
      - Diagnostics in observation space (e.g., Exercise 5)
      - $P^a(t_k)$: check positive definiteness & symmetry
  - Application & enhancement of “D. Diagnostic Component”
    - Comparing Forecast(background) & analysis with respect to truth:
      - $|x_i^{NoDA}(t_k)-x_i^t(t_k)|$, $|x_i^b(t_k)-x_i^t(t_k)|$, $|x_i^a(t_k)-x_i^t(t_k)|$, $\sqrt{P_{ii}^b(t_k)}$, and $\sqrt{trP^b(t_k)}$ (for specific i’s) vs $t_k$
      - $|x^{NoDA}(t_k)-x^t(t_k)|$, $|x^b(t_k)-x^t(t_k)|$, $|x^a(t_k)-x^t(t_k)|$, $\sqrt{trP^b(t_k)}$, $\sqrt{trP^a(t_k)}$ vs $t_k$
    - Comparing the results: EKF vs 3DVar/OI
  - Examination of the effect of dynamically estimated $P^b(t_k)$, including inflation
    - Start from no inflation, gradually increase & vary for an ‘optimal’ value
  - Examination of the effect of the observing system (but $H=\text{const.}$ for each experiment)